

THE LOCKER PROBLEM

Instructions. Work on the locker problem described below. As you work on the locker problem, have one member of your group keep track of information in a way that will allow you report on your progress.¹

Your report about your work on this problem should address the following areas:

- **Collaborators:** Who is in your group?
- **Understanding the Problem:** Describe any things you did to make sure *everyone* in the group understands the problem. Also list any misunderstandings you may have had.
- **Ideas/Questions:** Keep track of what you do (and think about doing) to solve this problem. Make a list of each idea that you consider using to solve the problem. For each say why you decided to pursue or not pursue your idea, and if you do pursue it, record what comes of it (good, bad or otherwise). Be sure to include ideas that someone suggested but that you did not pursue along with a reason why you did not pursue them.

Note: often your ideas can best be expressed as a question: What would happen if...? Do you think that...? Can we figure out...?

- **Conjectures:** Make a list of things that you think might be true but for which you do not have a convincing argument. Include with this the *evidence* that makes you believe your conjecture. (Include conjectures even if you change your mind later; just note that you changed your mind and why.)
- **Findings:** Make a list of things that you are able to show are true or false. Include a convincing argument for each finding.
- **Looking Back:** Whether you solve the problem “completely” or not, look back over your work periodically. Ask yourselves questions like: Have we overlooked anything? Do we have a goal? Are we making progress toward our goal? Is our solution really “complete”?
- **Miscellaneous:** Make note of anything else that you discover about the problem, yourself, mathematics, etc., which is not covered in the headings above.

Now, here is THE LOCKER PROBLEM...

One night the janitors at a local high school were bored, so they decided to play a little game. They noticed that there were exactly 100 lockers at the school, all of which were closed. The first janitor went down the halls and opened every locker. Let’s call that round 1. Then the second janitor went down the halls and closed every second locker (lockers 2, 4, 6, . . . , 98, 100). Let’s call that round 2. In round 3, the first janitor went down the halls again and switched the door (opened it if had been closed, closed it if it had been open) of every third locker (lockers 3, 6, . . . , 96, 99). In round 4, the other janitor changed the door on every fourth locker (lockers 4, 8, 12, . . . , 96, 100). This process continued for 100 rounds. (In round 100 only locker 100 was changed.) Then both janitors went home.

Which lockers were open at the end of all this? Why?

¹To spread the work around a bit, you might like to appoint one person to be “secretary” during class and a different person to take responsibility for writing things up to turn in next time.