

Monday, January 26

Preparation

- Read Wade, *Blaise Pascal: An Apologist for Our Time*; and *Pascal's Wager*
- Do these problems on a *separate sheet of paper*.

1. We said in class that the margin of error depends on the sample size. This problem investigates this statement in more detail.

- a) Recall that the margin of error also depends on the sample proportion (\hat{p}). The worst case (i.e. largest margin of error) occurs when the proportion is 0.5, so assume that situation and work out the margin of error for each of the sample sizes below.

sample size (n)	margin of error		
	sample prop. = 0.5	sample prop. = 0.3	sample prop. = 0.1
100			
400			
1600			
6400			

- b) Add columns in your chart for the margin of error assuming a sample proportion of 0.3 and 0.1.
- c) What do you notice when you look at the completed table?
2. Consider the following silly game. You roll a 6-sided die. If you roll a 6, you get \$6. If you roll a 5 you get \$5. Otherwise you win nothing. What is the expected value of this game?
3. A Roulette wheel has 18 red, 18 black and 2 green slots. If you bet \$1 on green, what is the most the casino can afford to pay you if they want to come out ahead? Round in the casino's favor to the nearest dollar. What is the expected value of a \$1 bet on green given this payout.
4. Keno is another lottery game. In this game, the player chooses 10 different numbers between 1 and 80. Later the lottery commission selects 22 numbers. Tickets cost \$1 and prizes are determined based on the number of matches:

Matches	prize	probability
0	\$1 (free lottery ticket)	3840963/121199272
1	\$0	4311285/30299818
2	\$0	162966573/605996360
3	\$0	4260564/15149909
4	\$0	141663753/787795268
5	\$0	72168327/984744085
6	\$7	15146439/787795268
7	\$25	3147312/984744085
8	\$250	252909/787795268
9	\$2,500	3451/196948817
10	\$250,000	119/302998180

Determine the expected value of a lottery ticket. Can you tell for which kind of prize the lottery commission pays out the most money?

Solutions to Problems

5. We said in class that the margin of error depends on the sample size. This problem investigates this statement in more detail.
- a) Recall that the margin of error also depends on the sample proportion (\hat{p}). The worst case (i.e. largest margin of error) occurs when the proportion is 0.5, so assume that situation and work out the margin of error for each of the sample sizes below.

sample size (n)	margin of error		
	sample prop. = 0.5	sample prop. = 0.3	sample prop. = 0.1
100	10%	9.2 %	6%
400	5%	4.6%	3%
1600	2.5%	2.3%	1.5%
6400	1.25%	1.15%	0.75%

- b) Add columns in your chart for the margin of error assuming a sample proportion of 0.3 and 0.1.
- c) What do you notice when you look at the completed table?

Increasing the sample size by a factor of 4 decreases the margin of error by a factor of only 2. While it is reasonably easy to get margins of error within a few percentage points, it would take an enormous sample to get to with 0.1%, for example. (The Bureau of Labor Statistics does large samples each month to estimate unemployment with in approximately 0.2% (and a number of other things).

6. Consider the following silly game. You roll a 6-sided die. If you roll a 6, you get \$6. If you roll a 5 you get \$5. Otherwise you win nothing. What is the expected value of this game?

$$e.v = 0 \cdot \frac{4}{6} + 5 \frac{1}{6} + 6 \frac{1}{6} = \frac{1}{1} / 6 = 1.83$$

7. A Roulette wheel has 18 red, 18 black and 2 green slots. If you bet \$1 on green, what is the most the casino can afford to pay you if they want to come out ahead? Round in the casino's favor to the nearest dollar. What is the expected value of a \$1 bet on green given this payout.

$e.v = 0 \cdot \frac{18}{19} + x \frac{1}{19} = \frac{x}{19}$, where x is the payout for winning. To make money, the casino must have the expected payout be less than \$1, so they could choose x to be \$18. (At \$19 they break even. At \$20 they lose money.

8. Keno is another lottery game . . .

Determine the expected value of a lottery ticket. Can you tell for which kind of prize the lottery commission pays out the most money?

Let's expand the table as follows:

Matches	prize	probability	prize \times prob
0	\$1 (free lottery ticket)	3840963/121199272	0.0316913
1	\$0	4311285/30299818	0
2	\$0	162966573/605996360	0
3	\$0	4260564/15149909	0
4	\$0	141663753/787795268	0
5	\$0	72168327/984744085	0
6	\$7	15146439/787795268	0.1345850
7	\$25	3147312/984744085	0.0799018
8	\$250	252909/787795268	0.0802585
9	\$2,500	3451/196948817	0.0438058
10	\$250,000	119/302998180	0.0981854

The sum of the values in the last column is 0.468427, so the expected value is 46.8427 ¢. (Note that this is a bit worse for the ticket buyer than the Daily 3 and Daily 4 Lotteries we looked at in class.)

The individual values in the last column indicate the expected payout for each type of prize individually. The largest value is for the \$7 prize, so more money will need to be spent on all those little \$7 prizes than on any other prize.

One additional note: The expected value is actually a little bit lower than what we have here because the free lottery ticket isn't really worth \$1. Taking that into account would reduce the expected value by a bit more than 1.5 ¢.

Class

- 8:30–9:50: Devotions, Quiz, Pascal’s Wager, Homework Discussion
- 10:30–11:50: Discussion, Wrap-up

Looking Ahead

The final exam Tuesday will begin at **9:30 am**.

As part of that final, you are asked to write an essay on one of the following topics.

- How do science (here meaning natural sciences and mathematics, not the broader definition given by Kuyper) relate to religion?

We have read and heard about several different perspectives on this topic. Outline your own position, comparing and contrasting it to two or three of the positions given by Haarsma, Atkins, Gould, McKay, Fang, Bradley, Polkinghorne, or Hamming. (If your position is essentially the same as one of these, do not also choose it for the compare/contrast part.)

Things I will be looking for:

- how well you articulate your own position;
- how well you represent the other positions you consider;
- how well your position addresses the issues raised by the other positions you address;
- some consideration of mathematics (e.g., probability and/or statistics). You may also include other sciences (astronomy, biology, physics, etc.).

You do not have to articulate a position that agrees with mine or even one that is “Reformed”, but you should address the kinds of issues a Reformed Christians are interested in. That is, if your position is not “Reformed”, imagine you are trying to persuade someone who is.

- What is your position on the lottery and casinos?

Your answer should include such things as

- the position you think individuals should take regarding playing the lottery or casino games;
- the position you think the government should have in these matters. (Imagine that you are either a legislator who must make decisions, or simply a voter trying to pick a candidate – what would you look for in a position regarding gaming?)

As in the preceding essay option, I am not requiring you to have a particular opinion, but I would like to see that you deal with the kinds of issues that have been important in this class.

Your essay may be turned in either at the time of the exam or by 5 pm.

Tuesday, January 27: Final Exam

The final exam will begin at 9:30 am. Your essay may be turned in either at the time of the exam or by 5 pm.