

Step by Step Example of A Simple Harmonic Study (based on a two bus system) by Constantine Hatziadoniou

Problem: Consider the industrial system of Figure 1. The system consists of two buses IND1 and IND2 connected through a short 3-phase, 4-wire line. The system is supplied by the utility through a 69kV/13.8 kV transformer. A line-commutated power-converter is connected on bus IND2. Calculate the harmonic voltage and THD on Buses IND1 and IND2.

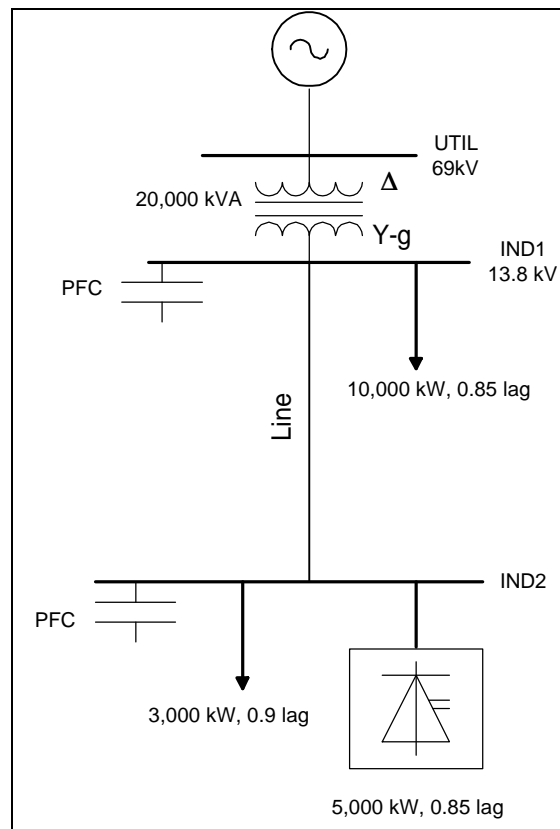


Fig. 1 Single line diagram of the example system.

The system data are as follows:

1. Utility: _____ 69 kV, infinite bus.
2. Transformer: _____ 69kV-Δ/13.8kV-Y-g, 20,000 kVA,
R=0.5%, X=8%.
3. Line: _____ Short distribution line
3-phase with ground wire:
Total positive sequence R= 0.02 Ω,
Total positive sequence reactance X= 0.06 Ω.
4. Load on IND1: _____ 10,000 kW, 0.85 lag pf.
Of this load, 60% is motive.

5. Load on IND2: _____ 3,000 kW, 0.9 lag pf.
Largely residential and commercial.
6. Converter on IND2: _____ 3-phase line commutated rectifier .
5,000 kW, 0.85 lag pf.
The rectifier produces the full spectrum of its characteristic orders at their normal amplitude and phase. Non-characteristic harmonic orders are not produced.
7. Power factor correction capacitors at IND1 and IND2: _____ Provide full compensation of the bus loads.

A. Discussion: The calculations will be done using the p.u. system equivalent. The base quantities are:

Power base=10,000 kVA

Impedance base (on the 13.8 kV side)=19Ω.

In this simple example, only positive and negative sequence harmonics will be considered. Thus, we will assume a symmetric and balanced system. Two methods of solution are presented and compared: Frequency domain using MatLab. Time domain using ATP. The associated files are available.

B. Load-Flow Study: The purpose of the load flow study is to obtain the fundamental frequency voltage magnitudes and phase angles. The former are used as basis to calculate THD. The latter are used to properly describe the phase-angle of the harmonic sources.

The system impedance is expressed in pu on the given basis. The system load-flow data are shown on Table 1. We assume that the utility voltage is 1 pu. The calculations to obtain the load-flow data including the pu system impedance are shown in the associated M-file. The load-flow solution is shown on Table 2. The load flow was solved using IPFLOW2.1.

Since in this system, only one harmonic source is present, the phase information from the load-flow solution is not critical. In addition, the system voltage is near 1.0 pu. Therefore we could have skipped the load-flow and proceeded to the harmonic calculations, using 1 pu as the base voltage for the THD calculation.

Table 1. Branch data.

From bus	To bus	R (pu)	X (pu)	Charging (pu)	Element Type
UTIL	IND1	0.0025	0.04	0	Transformer
IND1	IND2	0.0011	0.0032	0	Line

Table 2. Bus data.

Bus	P-gen	Q-gen	P-load	Q-load	Shunt	V	phase
UTIL	1.860	0.152	0	0	0	1.0	0.0
IND1	0	0	1.0	0.62	0.62	0.992	-4.16
IND2	0	0	0.8	0.455	0.455	0.991	-4.3

C. Harmonic Calculations in the Frequency Domain: The positive/negative sequence equivalent of the system harmonic impedance is shown in Figure 2 (h is the harmonic index). The system is balanced, therefore, the positive and negative sequence equivalents are identical. Thus, only one circuit is needed for the harmonic calculations.

The detail calculations are shown here. With reference to Figure 2, each device is modeled as follows:

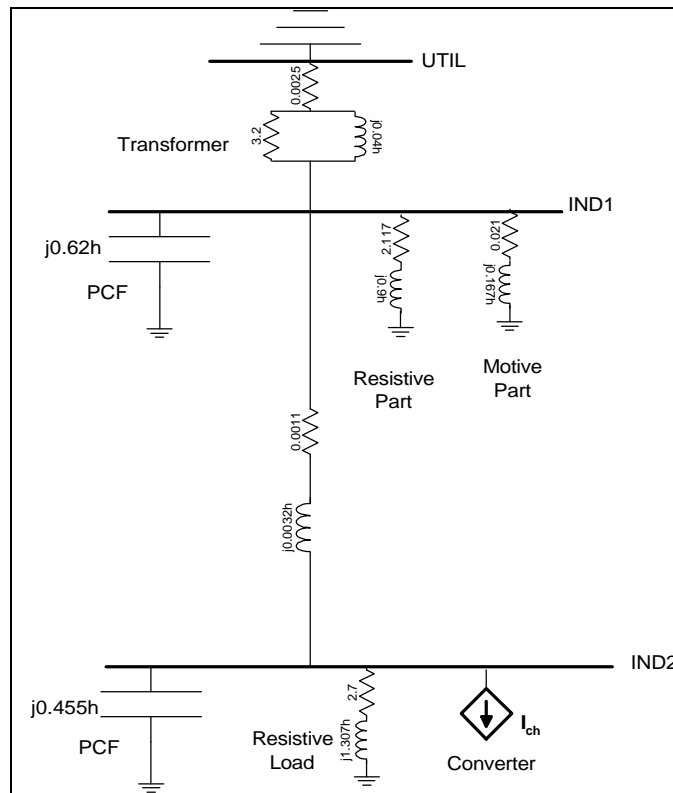


Fig. 2 Harmonic impedance diagram of the example system.

Utility system: The utility system is considered as infinite bus. Thus, it is a sinusoidal voltage source at fundamental frequency with zero impedance. At frequencies above the fundamental, the utility system is represented by a short circuit at Bus UTIL.

Transformer: The transformer is represented by a combination of series and parallel impedance between Buses UTIL and IND1. With reference to Figure 2, the values of

these elements, at the h^{th} harmonic order, are calculated directly from the fundamental-frequency series-impedance of the transformer:

Transformer series-impedance at fundamental: _____ $Z_T=R_T+jX_T=0.0025+j0.04$ pu.
 Series-resistance in the harmonic equivalent: _____ $R_s=R_T=0.0025$ pu.
 Parallel reactance: _____ $X_p=hX_T=0.04h$.
 Parallel resistance: _____ $R_p=80X_T=3.2$ pu.

Thus, the equivalent harmonic impedance of the transformer between UTIL and IND1 is given by (1).

$$Z_{Th} = R_s + \frac{h^2 X_T^2 R_p}{R_p^2 + h^2 X_T^2} + j \frac{h X_T R_p^2}{R_p^2 + h^2 X_T^2} \quad h > 1 \quad (1)$$

Transmission line: With reference to Figure 2, a simple representation is used for the line harmonic impedance. Since the line is of short length, the charging capacitance is neglected and only the series impedance is used (2).

$$Z_{Lh} = R_L + jhX_L = 0.0011 + j0.0032h \quad h \geq 1 \quad (2)$$

Where, R_L and X_L are, respectively, the pu series resistance and reactance of the line at fundamental frequency.

Equation (2) neglects the skin effect. This can be included by modifying the resistive part of the line impedance according to (3).

$$R_{Lh} = R_L \left[1 + \frac{0.646h^2}{192 + 0.518h^2} \right] \quad (3)$$

Load at IND1: The load of IND1 consists of 60% induction motors and of 40% resistive load (including coupling transformer reactance).

1. The motive part is represented by a series R-X circuit associated with locked-rotor impedance of the motor. This part is computed as follows:

Assume 0.8 lag as aggregate power factor for the motors.

Motor real power: _____ $P_M=0.6P_{IND1}=0.6$ pu.

P_{IND1} is the total load on the bus.

Motor installed power (apparent power): _____ $S_M=P_M/(0.8)=0.75$ pu.

Assume a severity factor $K_I=8$. The severity factor is the ratio between the starting and the rated current of the motor. From that, the combined pu locked-rotor reactance of the motive part at fundamental frequency is calculated (4):

$$X_M = \frac{V^2}{K_I S_M} = \frac{1}{8S_M} = 0.167 \text{ pu} \quad (4)$$

In (4) we assume 1 pu operating voltage for the motor.

The pu resistive part of the locked-rotor impedance is found by assuming a quality factor of 8 for the rotor circuit:

$$R_M = X_M/8 = 0.021 \text{ pu.}$$

Thus, the motor harmonic impedance in Figure 2 is given by (5).

$$Z_{Mh} = 0.021 + j0.167h \quad (5)$$

2. The resistive part at IND1 is represented by a series R-X impedance. The impedance elements are calculated from the power consumption of the load as follows:

Power of resistive load: _____ $P_R = 0.4 P_{IND1} = 0.4 \text{ pu.}$

Reactive power: _____ $Q_R = Q_{IND1} - Q_M = 0.62 - 0.6 S_M = 0.17 \text{ pu.}$

Installed power of resistive part: _____ $S_R = 0.435 \text{ pu at } 0.92 \text{ lag } (\phi_R = 23^\circ.03).$

The pu value of the fundamental impedance that produces the same power under 1 pu voltage is:

$$Z_R = 1/S_R = 2.3 \text{ pu.}$$

$$X_R/R_R = \tan(\phi_R) = 0.425.$$

Therefore: _____ $R_R = 2.117 \text{ pu, } X_R = 0.9 \text{ pu.}$

The load harmonic impedance at Bus IND1 is:

$$Z_{Rh} = 2.117 + j0.9h \quad (6)$$

Resistive load at IND2: Following the same procedure as with the load at IND1, the pu harmonic impedance of the load at IND2 is calculated as follows:

Load power and power factor: _____ $P_R = 0.3 \text{ pu @ } 0.9 \text{ lag } (\phi_R = 25^\circ.84).$

Installed power: _____ $S_R = P_R/0.9 = 0.333.$

PU fundamental impedance: _____ $Z_R = 1/S_R = 3 \text{ pu.}$

$$X_R/R_R = \tan(25^\circ.84).$$

$$R_R = 2.7 \text{ pu, } X_R = 1.307 \text{ pu.}$$

The load harmonic impedance is, therefore:

$$Z_{R2h} = 2.7 + j1.307h \quad (7)$$

Power factor correction capacitors: With reference to Figure 2, the PFCs are represented by their harmonic susceptance. The capacitor fundamental susceptance is found from the load flow data, since each capacitor fully compensates for the bus load.

Line commutated converter at IND2: With reference to Figure 2, the power converter is represented as a harmonic current source ejected from IND2. The converter produces the characteristic harmonic spectrum. Its fundamental current is found as follows:

Installed power: _____ $S_c = P/0.85 = 0.588$ pu.
 Fundamental current: _____ $I_{c1} = S_c / V_{IND2} = 0.594$ pu
 (from load-flow $V_{IND2} = 0.991$ pu).

The pu harmonic current of the converter is given in the following Table.

Table 3. Harmonic Currents of the line commutated converter.

h	5	7	11	13	17	19	23	25	29
%I _{c1}	0.2	0.143	0.091	0.077	0.059	0.053	0.043	0.04	0.034
I _{ch}	0.119	0.085	0.054	0.046	0.035	0.031	0.026	0.024	0.020
θ _h	-π	0	-π	0	-π	0	-π	0	-π

Table 3 (continued)

h	31	35	37	41	43	47	49
%I _{c1}	0.032	0.029	0.027	0.024	0.023	0.021	0.02
I _{ch}	0.019	0.017	0.016	0.014	0.014	0.012	0.012
θ _h	0	-π	0	-π	0	-π	0

C.1 Method of Calculation. The Nodal admittance matrix is used for the calculation of the system harmonic voltage. With reference to the circuit topology in Figure 2, at each frequency of the converter harmonic orders, this matrix is computed according to (8).

$$Y_h = \begin{bmatrix} \frac{1}{Z_{Th}} + \frac{1}{Z_{Lh}} + \frac{1}{Z_{Mh}} + \frac{1}{Z_{Rh}} + j0.62h & -\frac{1}{Z_{Lh}} \\ -\frac{1}{Z_{Lh}} & \frac{1}{Z_{Lh}} + \frac{1}{Z_{R2h}} + j0.455h \end{bmatrix} \quad (8)$$

Inverting the nodal admittance matrix we obtain the nodal impedance matrix at each harmonic order.

$$Z_h = Y_h^{-1} \quad (9)$$

Subsequently, the harmonic voltage at Bus IND1 (corresponding to row and column 1 of the matrices) is found using the transfer impedance, $Z_h(1,2)$, between Buses 2 and 1 (IND2 and IND1). The harmonic voltage at Bus IND2 (corresponding to row and column 2 of the matrices) is found using the driving point impedance, $Z_h(2,2)$, at Bus 2.

$$\begin{aligned} V_{IND1,h} &= Z_h(1,2)I_{ch} \\ V_{IND2,h} &= Z_h(2,2)I_{ch} \end{aligned} \quad (10)$$

Note that the converter current represents an injection at node 2 of the network.

Figure 3 shows the driving point and transfer impedance in (10). As it can be seen from the figure, the system forms two resonance frequency, one around the 5th harmonic and one around the 35th harmonic.

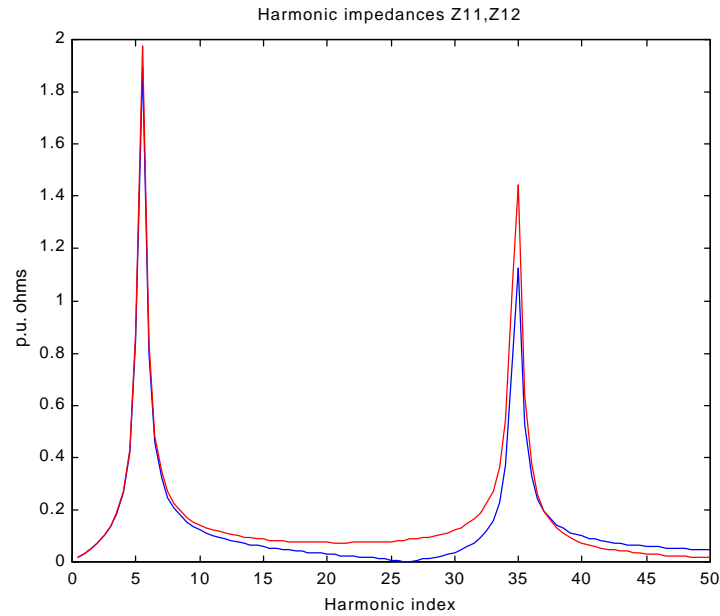


Fig. 3 System harmonic impedance.

Figure 4 shows the harmonic voltage at the two system buses. It can be seen from this figure that the harmonic voltage of both buses at the 5th and the 35th orders is amplified due to the impedance resonance.

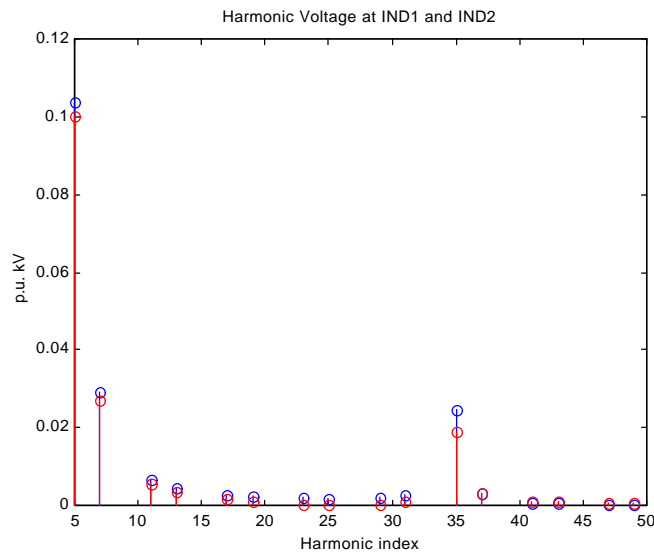


Fig. 4 System harmonic voltage.

D. EMTP-ATP Time-Domain Modeling. The purpose of the following calculations is to show how our harmonic problem can be solved using ATP modeling and simulation.

The positive/negative sequence equivalent-circuit used for the ATP simulation is shown in Figure 5. With reference to this figure, the circuit has identical topology as the circuit used for the frequency domain calculations.

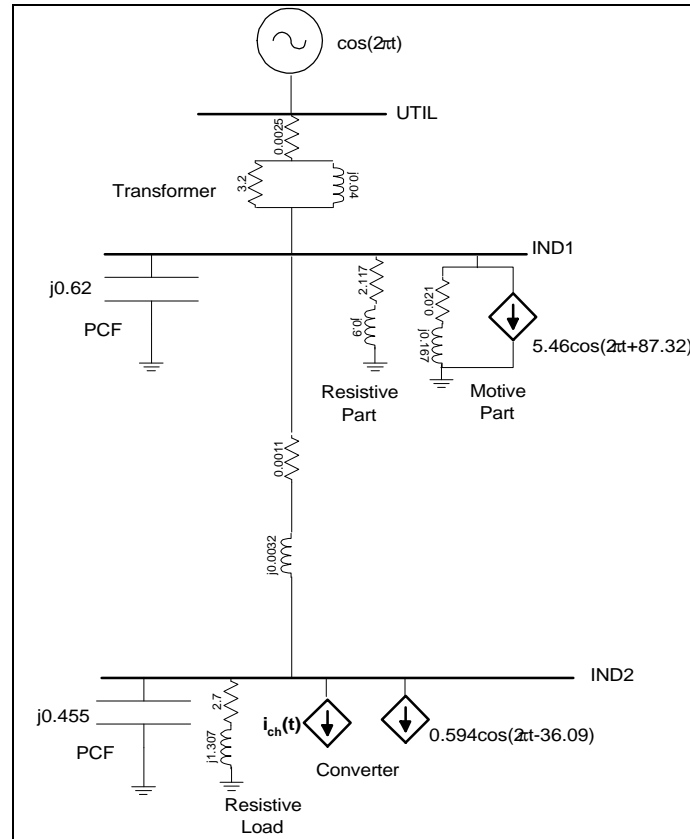


Fig. 5 ATP model of the example system (One phase simulation).

The circuit data are expressed in terms of inductance and capacitance. A harmonic current source is connected at the converter bus. Additional sources of fundamental frequency are included for the derivation and plotting of the system waveforms. It should be noted that the inclusion of these fundamental sources is not necessary for this example, since the circuit is linear. That is, the same response for the harmonic voltage would be obtained, if these sources were absent. Fundamental frequency sources are necessary, if accurate system waveforms are desired for demonstration purpose or if the circuit is non-linear. The calculation of the system data is as follows:

D.1. Calculation of Inductance and Capacitance. This would require converting the known pu values of the fundamental impedance to corresponding H of F. However, we have the alternative option of entering inductance and capacitance in pu. PU calculation of inductance and capacitance uses pu time. Here for convenience, the time base is set to 1/60 Hz. That is 1 pu time is equivalent to 1 fundamental period. The inductance and capacitance pu calculations are as follows:

$$v(t) = L \frac{di}{dt} = L \frac{di}{dt' t_B} = fL \frac{di}{dt'} = X_B \frac{X_L'}{2\pi} \frac{di}{dt'} \Rightarrow v'(t') = \frac{X_L'}{2\pi} \frac{di'}{dt'} \quad (11)$$

$$i(t) = C \frac{dv}{dt} = fC \frac{dv}{dt'} = B_B \frac{B_C'}{2\pi} \frac{dv}{dt'} \Rightarrow i'(t') = \frac{B_C'}{2\pi} \frac{dv'}{dt'} \quad (12)$$

Where: $t_B=1/f$ is the time base.

t' is the time in pu.

X_B, B_B are respectively the base of reactance and susceptance.

X_L', B_C' are respectively the pu fundamental reactance and susceptance of the inductor and capacitor respectively.

Thus, the pu inductance and capacitance are found from the pu fundamental reactance and susceptance respectively dividing by 2π . ATP data entry permits further simplification of these conversions. Setting the options XOPTION=COPTION=1.0 Hz tells ATP to accept the branch data as pu fundamental reactance or susceptance. This choice facilitates data entry, since no conversion of any sort is needed (ATP will perform the conversions from (11) and (12) internally.) Figure 5 shows the pu fundamental reactance and susceptance of the system as they were entered in the ATP data file. These values are retrieved directly from the previous frequency domain data.

D.2. Source Modeling. To obtain the accurate waveform of the voltage, the fundamental as well as the harmonic sources must be described with their correct angle. With reference to Figure 5, the external utility is modeled as a voltage source at UTIL with unity amplitude and zero angle. Note that the *cos* function is used for describing the sources. This is consistent with the ATP source modeling and also makes the calculation of harmonic phase angles easier.

The induction motor at Bus IND1 must be properly augmented by a fundamental current source, because the series R-X representation alone does not account for the motor's full fundamental current. Equation (13) is the current of the motor drawn at fundamental frequency as it is calculated from the load flow. Equation (14) is the current of the motor drawn at fundamental frequency as it is calculated by the circuit representation of the motor in Figure 5.

$$\overline{I_1} = S_M^* / \overline{V_M^*} \approx 0.75^{\angle -36.87} / 1^{\angle 4.16} = 0.75^{\angle -41.03} \text{ puA} \quad (13)$$

$$\overline{I_1} = \overline{V_M} / (R + jX) + \overline{I_M} = 1^{\angle -4.16} / (0.021 + j0.167) + \overline{I_M} = 5.95^{\angle -87} + \overline{I_M} \quad (14)$$

Where: I_M is the augmented current source. The phase angles are obtained from the load flow solution and the power factor. Equating (13) and (14) we obtain the value of the augmented current source in Figure 5.

$$\overline{I_M} = \overline{I_1} - \overline{I_1} = 5.46^{\angle 87.32}$$

The converter at Bus IND2 is modeled as a combination of a fundamental and a harmonic current source. The fundamental current of the converter in Figure 5 is calculated from the load flow solution, the phase angle of the bus voltage and the power factor of the converter. The value of the harmonic source is calculated using Table 3. Thus, the phase angle of the n^{th} harmonic is found according to (15).

$$y_n = y_1 + n\theta_n \quad (15)$$

Where: ψ_1 is the phase angle of the fundamental current,
 θ_n is the harmonic angle relative to the fundamental current from Table 3.

Table 4 shows the description of the equivalent harmonic current sources as they are included in the ATP file. (Note: time is in pu.)

Table 4 Equivalent current sources at IND2.

Harmonic Order	Description
5	0.119cos(5t-0.45)
7	0.085cos(7t+107.37)
11	0.054cos(11t+143)
13	0.046cos(13t-109.17)
17	0.035cos(17t-73.53)
19	0.031cos(19t+34.29)
23	0.026cos(23t+69.93)
25	0.024cos(25t-112.32)
29	0.020cos(29t-146.61)
31	0.019cos(31t-38.70)
35	0.017cos(35t-3.15)
37	0.016cos(37t+104.67)
41	0.014cos(41t+140.31)
43	0.014cos(43t-111.87)
47	0.012cos(47t-76.23)
49	0.012cos(49t+31.59)

Figure-6 shows the waveform of the system voltage. Figure-7 shows the harmonic content of that voltage.

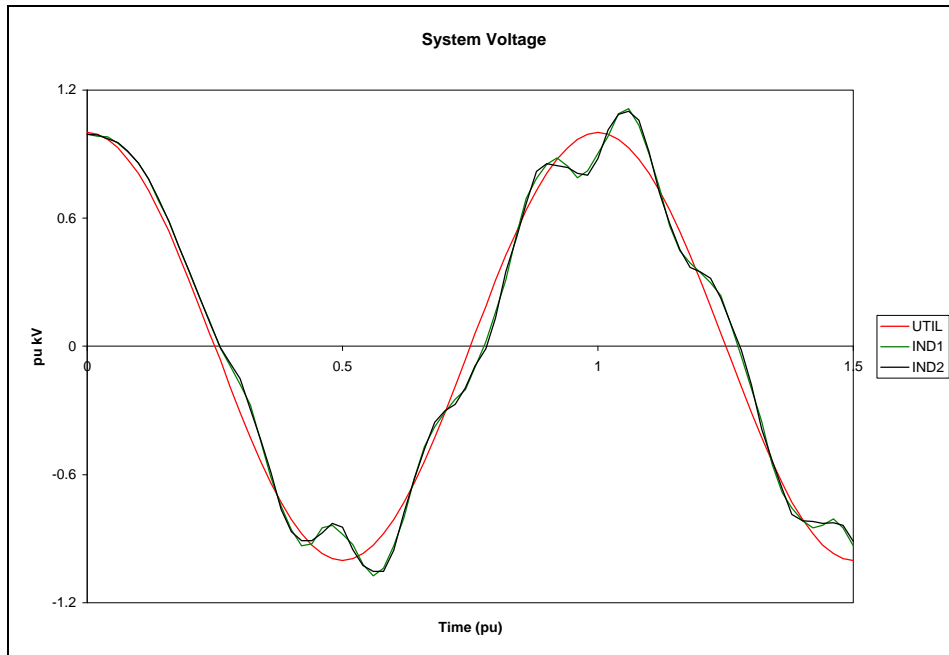


Fig. 6 Wave form of harmonic voltage.

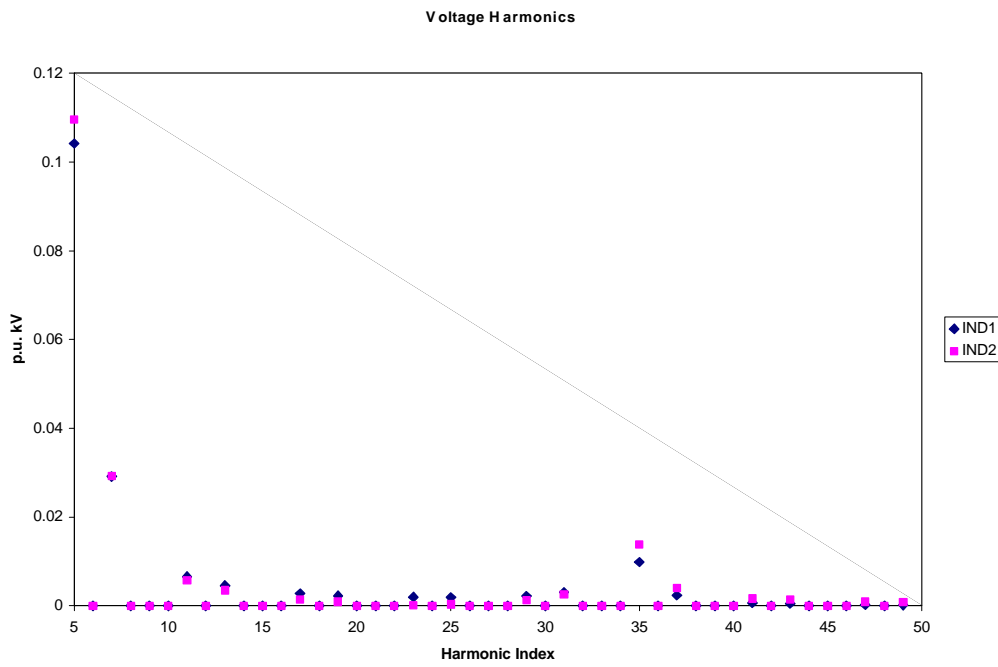


Fig. 7 Harmonic content of system voltage.