HARMONICS THEORY

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2.1 Introduction

For most conventional analyses, the power system is essentially modeled as a linear system with passive elements excited by constant- magnitude and constant-frequency sinusoidal voltage sources. However, with the widespread proliferation of power electronics loads nowadays, significant amounts of harmonic currents are being injected into power systems. Harmonic currents not only disturb loads that are sensitive to waveform distortion, but also cause many undesirable effects on power system elements. As a result, harmonic studies are becoming a growing concern.

Harmonics are usually defined as periodic steady state distortions of voltage and/or current waveforms in power systems. In the harmonic polluted environment, the theory regarding harmonic quantities needs to be defined to distinguish from those quantities defined for the fundamental frequency.

The purpose of this chapter is to present basic harmonics theory. Initially, the Fourier series and analysis methods that can be used to interpret waveform phenomenon are reviewed. Some fundamentals of Fourier transforms used in today's harmonics measurement techniques are also introduced. The general harmonics theory, the definitions of harmonic quantities, harmonic indices in common use, and power system response and solutions to harmonics are then described.

2.2 Fourier Series and Analysis

The theory of the Fourier series was first introduced by the French physicist and mathematician, Joseph Fourier, in his article 'Analytic Theory of Heat' which was published in 1882. The theory involves expansions of arbitrary functions in certain types of trigonometric series. It proves that any periodic function in an interval of time could be represented by the sum of a fundamental and a series of higher orders of harmonic components at frequencies which are integral multiples of the fundamental component. The series establishes a relationship between the function in time and frequency domains. Today, the theory has become the famous 'Fourier series' and it is one of the most important tools for engineers and scientists in many applications.

Fourier Series

A periodic function can be defined as any function for which

$$f(t) = f(t+T) \qquad (2.1)$$

for all t. The smallest constant T that satisfies (2.1) is called the period of the function. By iteration of (2.1), we have

$$f(t) = f(t+hT), \quad h = 0,\ \pm 1,\ \pm 2,\ \ldots\ . \qquad (2.2)$$

Let a function *f(t)* be periodic with period *T*, then this function can be represented by the trigonometric series

$$f(t) = \frac{1}{2}a_0 + \sum_{h=1}^{\infty}\{a_h \cos(h\omega_0 t) + b_h \sin(h\omega_0 t)\}, \qquad (2.3)$$

where $\omega_0 = 2\pi / T$. A series such as (2.3) is called trigonometric Fourier series. It can be rewritten as

$$f(t) = c_0 + \sum_{h=1}^{\infty} c_h \sin(h\omega_0 t + \phi_h), \qquad (2.4)$$

where $c_0 = a_0/2$, $c_h = \sqrt{a_h^2 + b_h^2}$, and $\phi_h = \tan^{-1}(a_h / b_h)$.

Observing (2.4), we see that the Fourier series expression of a periodic function represents a periodic function as a sum of sinusoidal components with different frequencies. The component of $h\omega_0$ is called the *h*-th harmonic of the periodic function. c_0 is the magnitude of the dc component. The component with $h = 1$ is called the fundamental component. c_h and ϕ_h are known as the *h*-th order harmonic magnitude and phase angle, respectively. The magnitude and phase angle of each harmonic determine the resulting waveshape of *f(t)*.

Equation (2.3) also can be represented by its complex form as

$$f(t) = \sum_{h=-\infty}^{\infty} c_h e^{jh\omega_0 t}, \qquad (2.5)$$

where for $h = 0,\ \pm 1,\ \pm 2,\ \ldots$,

$$c_h = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jh\omega_0 t} dt\ . \qquad (2.6)$$

Orthogonal Functions

A set of functions $\{\varphi_h(t)\}$ is called orthogonal on an interval $\alpha < t < \beta$ if all groups of any two functions $\varphi_i(t)$ and $\varphi_j(t)$ in the set $\{\varphi_h(t)\}$ satisfy

$$\int_{\alpha}^{\beta} \varphi_i(t)\varphi_j(t)dt = \begin{cases} 0, & i \neq j \\ \gamma, & i = j \end{cases}, \qquad (2.7)$$

where γ is a nonzero value. It can be shown that $\{1, \cos\omega_0 t, \ldots, \cos h\omega_0 t, \ldots, \sin\omega_0 t, \ldots, \sin h\omega_0 t, \ldots\}$ is an orthogonal set of sinusoidal functions on interval $-T/2 < t < T/2$. Using the orthogonal relations, we can show that Fourier coefficients a_0, a_h, and b_h of (2.3) are

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt\,, \tag{2.8}$$

$$a_h = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(h\omega_0 t)dt, \text{ and} \tag{2.9}$$

$$b_h = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(h\omega_0 t)dt, \tag{2.10}$$

where $h = 1, 2, \ldots$.

For the set of complex valued functions $\{\varphi_h(t)\}$, it can be shown that (2.7) holds when $\varphi_j(t)$ is the complex conjugate of $\varphi_i(t)$.

Waveform Symmetry

A function *f(t)* is called an even function if it has the property

$$f(-t) = f(t), \tag{2.11}$$

and it is called an odd function if

$$f(-t) = -f(t). \tag{2.12}$$

An even function is symmetrical to the vertical axis at the origin, and an odd function is anti-symmetrical to the vertical axis at the origin. A function with a period of T is half-wave symmetry if it satisfies the condition

$$f(t) = -f(t \pm T/2). \tag{2.13}$$

If *f(t)* has half-wave symmetry and is either an even or odd function, then it has even or odd quarter-wave symmetry. The use of symmetry simplifies the calculation of Fourier coefficients in (2.8) - (2.10).

Fourier Transform

The Fourier transform of a function *f(t)* is defined as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\,, \tag{2.14}$$

and *f(t)* is called the inverse Fourier transform of $F(\omega)$, which is defined as

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega\ . \tag{2.15}$$

Equations (2.14) and (2.15) are often called the Fourier transform pair, and they are used to map any function in the interval of $(-\infty, \infty)$ in time or frequency domain into a continuous function in the inverse domain. The key property of the Fourier transform is its ability to examine a function or waveform from the perspective of both the time and frequency domains. A given function can have two equivalent modes of representations: one is in the time domain and is called *f(t)*, and the other is in the frequency domain and is called $F(\omega)$. Equation (2.14) transforms the time function into a frequency spectrum, and (2.15) synthesizes the frequency spectrum to regain the time function.

Discrete Fourier Transform

When the frequency domain spectrum and the time domain function are both periodic sampled functions with N samples per period, (2.14) and (2.15) can be represented by the following so-called discrete Fourier transform (DFT) pair:

$$F(k\Delta\Omega) = \sum_{n=0}^{N-1} f(n\Delta T)e^{-j2\pi kn/N}\,, \tag{2.16}$$

and

$$f(n\Delta T) = \sum_{k=0}^{N-1} F(k\Delta\Omega)e^{j2\pi kn/N}\,, \tag{2.17}$$

where k, n = 0, 1, ..., N-1, $\Delta\Omega = 2\pi/\Delta T$ and $\Delta T = T/N$. The DFT is often used in harmonic measurement because the measured data is always available in the form of a sampled time function. The sampled time function is represented by a time series of points of known magnitude separated by fixed time intervals of limited duration.

Fourier analysis can be done by DFTs. The DFTs are often calculated by the use of fast Fourier transform (FFT) algorithm [1]. FFT techniques are very fast methods for performing the DFT calculations (2.16) and (2.17) which allow the evaluation of a large number of functions. There are a number of available FFT algorithms that can be easily used in harmonic analysis.

2.3 Basic Definitions of Harmonic Quantities

Conventionally, the definitions used to describe electric quantities for power system study are for systems operating in sinusoidal steady state. However, when harmonics are present because of system nonlinearities, the definitions of these electric quantities need to be modified from those appropriate for single-frequency systems.

This section gives the basic definition of power system harmonics and describes some useful definitions associated with voltage, current, instantaneous power, average (active) power, apparent power, reactive power, and power factor computations under nonsinusoidal situations.

Definition of Power System Harmonics

In the power system, the definition of a harmonic can be stated as: A sinusoidal component of a periodic wave having a frequency that is an integral multiple of the fundamental frequency. Thus for a power system with f_0 fundamental frequency, the frequency of the h-th order of harmonic is hf_0. Harmonics are often used to define distorted sinewaves associated with currents and voltages of different amplitudes and frequencies.

One can compose a distorted periodic waveshape of any conceivable shape by using different harmonic frequencies with different amplitudes. Conversely, one can also decompose any distorted periodic waveshape into a fundamental wave and a set of harmonics. This decomposition process is called Fourier analysis. With this technique, we can systematically analyze the effects of nonlinear elements in power systems.

Most elements and loads in a power system respond the same in both positive and negative half-cycles. The produced voltages and currents have half-wave symmetry. Therefore, harmonics of even orders are not characteristic. Also, triplens (multiples of third harmonic) always can be blocked by using three-phase ungrounded-wye or delta transformer connections in a balanced system, because triplens are entirely zero sequence. For these reasons, even-ordered and triplens are often ignored in harmonic analysis. Generally, the frequencies of interests for harmonic analysis are limited to the 50th multiple.

One major source of harmonics in the power system is the static power converter. Under ideal operating conditions, the current harmonics generated by a *p*-pulse line-commutated converter can be characterized by $I_h = I_1 / h$ and $h = pn \pm 1$ (characteristic harmonics) where n = 1, 2, ... and p is an integral multiples of six. If 1) the converter input voltages are unbalanced or 2) unequal commutating reactances exist between phases or 3) unequally spaced firing pulses are present in the converter bridge, then the converter will produce non-characteristic harmonics in addition to the characteristic harmonics. Non-characteristic harmonics are those that are not integer multiples of the fundamental power frequency.

The harmonic frequencies that are not integral multiples of the fundamental power frequency are usually called interharmonics. A major source of interharmonics is the cycloconverter [2]. One special subset of interharmonics is called sub-harmonics. Sub-harmonics have frequency values that are less than that of the fundamental frequency. Lighting flicker is one indication of the presence of sub-harmonics. A well-known source of flicker is the arc furnace [3].

Electric Quantities Under Nonsinusoidal Situation

When steady-state harmonics are present, instantaneous voltage and current can be represented by Fourier series as follows:

$$v(t) = \sum_{h=1}^{\infty} v_h(t) = \sum_{h=1}^{\infty} \sqrt{2} V_h \sin(h\omega_0 t + \theta_h), \quad (2.18)$$

$$i(t) = \sum_{h=1}^{\infty} i_h(t) = \sum_{h=1}^{\infty} \sqrt{2} I_h \sin(h\omega_0 t + \delta_h), \quad (2.19)$$

where the dc terms are usually ignored for simplicity, V_h and I_h are rms values for *h*-th order of harmonic voltage and current, respectively.

The instantaneous power is defined as

$$p(t) = v(t) i(t), \quad (2.20)$$

and the average power over one period T of $p(t)$ is defined as

$$P = \frac{1}{T}\int_0^T p(t)dt \, . \quad (2.21)$$

If we substitute (2.18) and (2.19) into (2.20) and make use of the orthogonal relations of (2.7), it can be shown that

$$P = \sum_{h=1}^{\infty} V_h I_h \cos(\theta_h - \delta_h) = \sum_{h=1}^{\infty} P_h \, . \quad (2.22)$$

We see that each harmonic makes a contribution, either plus or minus, to the average power. There are no contributions to the average power from the voltage at one frequency and the current at another. The average power generated by harmonics is usually very small in comparison with the fundamental average power.

By applying orthogonal relations, the rms values of (2.18) and (2.19) are proved to be

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\sum_{h=1}^{\infty} V_h^2} \, , \quad (2.23)$$

and

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\sum_{h=1}^{\infty} I_h^2} \, , \quad (2.24)$$

respectively.

The apparent power is given by

$$S = V_{rms} I_{rms} . \quad (2.25)$$

A widely accepted definition of apparent power is

$$S^2 = P^2 + Q^2 + D^2, \quad (2.26)$$

where Q is the reactive power defined as

$$Q = \sum_{h=1}^{\infty} V_h I_h \sin(\theta_h - \delta_h), \quad (2.27)$$

and D is defined as the distortion voltamperes which correspond to the products of voltages and currents of different frequency components in (2.18) and (2.19).

When harmonics are not present in (2.25), S is equal to $V_1 I_1$ which is the conventionally defined apparent power at fundamental frequency. Under the sinusoidal situation, the power equation relates mutually the average, reactive, and apparent power, and it is defined as

$$(V_1 I_1)^2 = P_1^2 + Q_1^2, \quad (2.28)$$

where $Q_1 = V_1 I_1 \sin(\theta_1 - \delta_1)$ is the fundamental reactive power defined in (2.27) for h = 1.

At present, there is still no consensus in the definitions and physical meanings regarding reactive power and distortion power among researchers and scientists [4-7]. In [8], some alternate definitions with interpretations on power definitions other than the above are described.

The concept of power factor originated from the need to quantify how efficiently a load utilizes the current that it draws from the ac power system. Regardless of sinusoidal or nonsinusoidal situation, the total power factor is defined as

$$pf = \frac{P}{S} \, , \quad (2.29)$$

where P is the average power contributed by the fundamental frequency component and other harmonic components, as shown in (2.22). In the next section, we also will show the relationship between the power factor and some harmonic distortion indices.

Phase Sequences of Harmonics

For a three-phase balanced system under nonsinusoidal conditions, the h-th order of harmonic voltage of each phase can be expressed as

$$v_{ah}(t) = \sqrt{2}V_h \sin(h\omega_0 t + \theta_h), \tag{2.30}$$

$$v_{bh}(t) = \sqrt{2}V_h \sin(h\omega_0 t - 2h\pi/3 + \theta_h), \tag{2.31}$$

$$v_{ch}(t) = \sqrt{2}V_h \sin(h\omega_0 t + 2h\pi/3 + \theta_h). \tag{2.32}$$

Therefore, the harmonic phase sequence in a balanced three-phase system has the pattern shown in Table 1.1.

Table 1.1. Harmonic Phase Sequences in a Balanced Three-Phase Power System

Harmonic Order	Phase Sequence
1	+
2	-
3	0
4	+
5	-
6	0
.	.

Observing Table 1.1, we find that the negative and zero sequences are also present in the system, and all triplens are entirely zero sequence. The above simple phase sequence pattern does not hold for the unbalanced system, because harmonics of each order contain the three different sequences. It requires a more complicated analysis [9].

The definitions in (2.18) - (2.24) are also suitable for three-phase balanced system. However, for the unbalanced system, the apparent power needs to be redefined and the consensus has yet to be reached. Reference [10] provides some practical power definitions under unbalanced conditions.

2.4 Harmonic Indices

In harmonic analysis there are several important indices used to describe the effects of harmonics on power system components and communication systems. This section describes the definitions of those harmonic indices in common use [11-13].

Total Harmonic Distortion (Distortion Factor)

The most commonly used harmonic index is

$$THD_V = \frac{\sqrt{\sum_{h=2}^{\infty} V_h^2}}{V_1} \quad \text{or} \quad THD_I = \frac{\sqrt{\sum_{h=2}^{\infty} I_h^2}}{I_1}, \tag{2.33}$$

which is defined as the ratio of the rms value of the harmonic components to the rms value of the fundamental component and usually expressed in percent. This index is used to measure the deviation of a periodic waveform containing harmonics from a perfect sinewave. For a perfect sinewave at fundamental frequency, the THD is zero. Similarly, the measures of individual harmonic distortion for voltage and current at h-th order are defined as V_h/V_1 and I_h/I_1, respectively.

Total Demand Distortion

The total demand distortion (TDD) is the total harmonic current distortion defined as

$$TDD = \frac{\sqrt{\sum_{h=2}^{\infty} I_h^2}}{I_L}, \tag{2.34}$$

where I_L is the maximum demand load current (15- or 30-minute demand) at fundamental frequency at the point of common coupling (PCC), calculated as the average current of the maximum demands for the previous twelve months. The concept of TDD is particularly relevant in the application of IEEE Standard 519.

Telephone Influence Factor

Telephone influence factor (*TIF*) is a measure used to describe the telephone noise originating from harmonic currents and voltages in power systems. *TIF* is adjusted based on the sensitivity of the telephone system and the human ear to noises at various frequencies. It is defined as

$$TIF_V = \frac{\sqrt{\sum_{h=1}^{\infty} (w_h V_h)^2}}{V_{rms}} \quad \text{or} \quad TIF_I = \frac{\sqrt{\sum_{h=1}^{\infty} (w_h I_h)^2}}{I_{rms}}, \tag{2.35}$$

where w_h is a weighting accounting for audio and inductive coupling effects at the h-th harmonic frequency. Obviously, *TIF* is a variation of the previously defined *THD* where the root of the sum of the squares is weighted using factors that reflect the response in the voice band.

V·T and I·T Products

Another distortion index that gives a measure of harmonic interference on audio circuits similar to *TIF* is the *V·T* or *I·T* product, where *V* is rms voltage in volts, *I* is rms current in amperes, and *T* is the *TIF*. In practice, telephone interference is often expressed as *V·T* or *I·T*, which is defined as

$$V \cdot T = \sqrt{\sum_{h=1}^{\infty} (w_h V_h)^2} \quad \text{or} \quad I \cdot T = \sqrt{\sum_{h=1}^{\infty} (w_h I_h)^2}, \tag{2.36}$$

where w_h is the same as previously described. If *kV·T* or *kI·T* is used, then the index must be multiplied by a factor of 1000. Equation (2.36) refers to the fact that the index is a product of harmonic voltage or harmonic current and the corresponding telephone influence factor. Observing (2.35) and (2.36), we find that

$$TIF_V \cdot V_{rms} = V \cdot T \quad \text{and} \quad TIF_I \cdot I_{rms} = I \cdot T. \tag{2.37}$$

C-Message Weighted Index

The *C*-message weighted index is similar to *TIF*, except that each weighting c_h is used in place of w_h. The weighting is derived from listening tests to indicate the relative annoyance or speech impairment by an interfering signal of frequency f as heard through a "500-type" telephone set. This index is defined as

$$C_V = \frac{\sqrt{\sum_{h=1}^{\infty}(c_h V_h)^2}}{V_{rms}} \quad \text{or} \quad C_I = \frac{\sqrt{\sum_{h=1}^{\infty}(c_h I_h)^2}}{I_{rms}}. \qquad (2.38)$$

The relation between *TIF* weight and *C*-message weight is

$$w_h = 5 c_h f_h, \qquad (2.39)$$

where f_h is the frequency of the *h*-th order harmonic.

Transformer K-Factor

Transformer K-factor is an index used to calculate the derating of standard transformers when harmonic currents are present [14]. The K-factor is defined as

$$K = \frac{\sum_{h=1}^{\infty} h^2 (I_h / I_1)^2}{\sum_{h=1}^{\infty} (I_h / I_1)^2}, \qquad (2.40)$$

where h is the harmonic order and I_h / I_1 is the corresponding individual harmonic current distortion. (2.40) is calculated based on the assumption that the transformer winding eddy current loss produced by each harmonic current component is proportional to the square of the harmonic order and the square of magnitude of the harmonic component.

The K-rated transformer is constructed to withstand more voltage distortion than standard transformers. The K-factor actually relates to the excessive heat that must be dissipated by the transformer. It is considered in the design and installation stage for nonlinear loads, and it is used as a specification for new or replacement power source equipment. Table 2 shows typical commercially available K-rated transformers, where all regular transformers fall into K-1 category.

Table 2: Commercially Available K-Rated Transformers

Category
K-4
K-9
K-13
K-20
K-30
K-40

Distortion Power Factor

When voltage and current contain harmonics, it can be shown [15] that

$$V_{rms} = V_1 \sqrt{1 + (THD_V / 100)^2} \qquad (2.41)$$

and

$$I_{rms} = I_1 \sqrt{1 + (THD_I / 100)^2} \qquad (2.42)$$

by substituting (2.23) and (2.24) into (2.33). The total power factor in (2.29) becomes

$$pf_{tot} = \frac{P}{V_1 I_1 \sqrt{1 + (THD_V / 100)^2}\sqrt{1 + (THD_I / 100)^2}}. \qquad (2.43)$$

In most cases, only very small portion of average power of P is contributed by harmonics and total harmonic voltage distortion is less than 10%. Thus (2.43) can be expressed as

$$pf_{tot} \approx \frac{P_1}{V_1 I_1} \cdot \frac{1}{\sqrt{1 + (THD_I / 100)^2}} = \cos(q_1 - d_1) \cdot pf_{dist}. \qquad (2.44)$$

In (2.44), the first term, $\cos(q_1 - d_1)$, is known as the displacement power factor, and the second term, pf_{dist}, is defined as the distortion power factor. Because the displacement power factor is always not greater than one, we have

$$pf_{tot} \leq pf_{dist}. \qquad (2.45)$$

Obviously, for single-phase nonlinear loads with high current distortion, the total power factor is poor. It also should be noted that adding power factor correction capacitors to such load is likely to cause resonance conditions. An alternative to improve the distortion power factor is using passive or active filter to cancel harmonics produced by nonlinear loads.

2.5 Power System Response to Harmonics

In comparison with the load, a power system is stiff enough to withstand considerable amounts of harmonic currents without causing problems. This means that the system impedance is smaller compared to the load impedance. A power system itself is not a significant source of harmonics. However, it becomes a contributor of problems by way of resonance when severe distortion exists.

Assuming all nonlinear loads can be represented as harmonic current injections, the harmonic voltage at each bus in a power system can be obtained by solving the following impedance matrix or nodal admittance equations for all orders of harmonics under consideration:

$$\mathbf{V_h} = \mathbf{Z_h} \cdot \mathbf{I_h} \qquad (2.46)$$

or

$$\mathbf{I_h} = \mathbf{Y_h} \cdot \mathbf{V_h}, \qquad (2.47)$$

where $\mathbf{V_h}$ is the vector consisting of the *h*-th harmonic voltage at each bus that is to be determined. $\mathbf{Z_h}$ is the system harmonic impedance matrix, $\mathbf{Y_h}$ is the system harmonic admittance matrix, and $\mathbf{I_h}$ is the vector of measured or estimated harmonic currents representing the harmonic-generating loads at connected busses.

In (2.46), $\mathbf{Z_h}$ can be obtained by using a *Z*-bus building algorithm for each harmonic of interest or from the inverse of $\mathbf{Y_h}$ in (2.47), but the harmonic effects on different power system components and loads need to be properly modeled [16]. Approaches for harmonic analysis based on (2.46) or

(2.47) are commonly called current injection methods. These approaches are usually used in conjunction with fundamental frequency load flow computations. Through providing the network harmonic impedance or admittance and harmonic currents injected by nonlinear loads for all harmonics under consideration, the individual and total harmonic voltage distortions at each bus can be determined. Reference [16] also describes some other harmonic analysis methods.

Observing (2.46), we see that system harmonic impedance plays an important role in the system response to harmonics, especially when resonance occurs in the system. Resonance is defined as an amplification of power system response to a periodic excitation when the excitation frequency is equal to a natural frequency of the system. For a simple LC circuit excited by a harmonic current, the inductive and capacitive reactance seen from the harmonic current source are equal at the resonant frequency $f_r = 1/(2\pi\sqrt{LC})$.

In a power system, most significant resonance problems are caused by a large capacitor installed for displacement power factor correction or voltage regulation purposes. The resonant frequency of the system inductive reactance and the capacitor reactance often occurs near fifth or seventh harmonic. However, resonant problems occurring at eleventh or thirteenth harmonic are not unusual. There are two types of resonances likely to occur in the system: series and parallel resonance. Series resonance is a low impedance to the flow of harmonic current, and parallel resonance is a high impedance to the flow of harmonic current.

Series Resonance

As shown in Figure 2.1, if the capacitor bank is in series with the system reactance and creates a low impedance path to the harmonic current, a series resonance condition may result. Series resonance may cause high voltage distortion levels between the inductance and the capacitor in the circuit due to the harmonic current concentrated in the low impedance path it sees. Series resonance often causes capacitor or fuse failures because of overload. The series resonant condition is given by

$$h_r = \sqrt{\frac{X_C}{X_L}}, \tag{2.48}$$

where h_r is the harmonic order of resonant frequency.

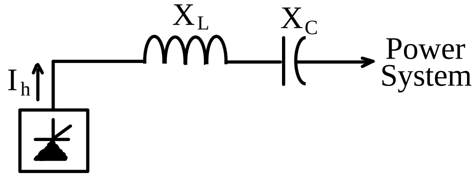

Figure 2.1. Series Resonance

Parallel Resonance

Figure 2.2 shows the circuit topology in which parallel resonance is likely to occur. Parallel resonance occurs when the parallel inductive reactance and the parallel capacitive reactance of the system are equal at certain frequency, and the parallel combination appears to be a very large impedance to the harmonic source. The frequency where the large impedance occurs is the resonant frequency. When parallel resonance exists on the power system, significant voltage distortion and current amplification may occur. The highly distorted bus voltage may cause distorted currents flowing in adjacent circuits. The amplified current may result in equipment failure.

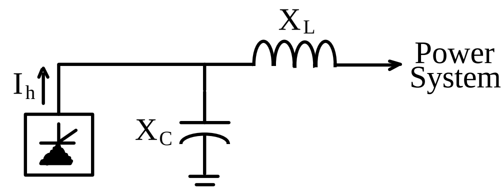

Figure 2.2. Parallel Resonance

When parallel resonance occurs in the circuit of Figure 2.2, the resonant frequency can be determined by

$$h_r = \sqrt{\frac{X_C}{X_L}} = \sqrt{\frac{MVA_{SC}}{MVAR_{CAP}}}, \tag{2.49}$$

where MVA_{SC} is the short-circuit MVA at the harmonic-generating load connection point to the system and $MVAR_{CAP}$ is MVAR rating of the capacitor. It should be understood that this approximation is only accurate for systems with high X/R ratios.

Another resonant scheme is shown in the distribution network of Figure 2.3. If some of the feeder inductance appears between groups of smaller capacitor banks, the system may present a combination of many series and parallel resonant circuits, although the resonant effects are somewhat less than that caused by one large resonant element. For this type of resonance problem, more sophisticated harmonic analysis programs must be employed to predict the harmonic characteristics of the system.

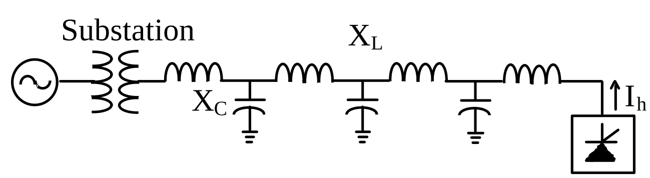

Figure 2.3. Distributed Resonance

2.6 Solutions to Harmonics

Passive harmonic filters are an effective mitigation method for harmonic problems. The passive filter is generally designed to provide a path to divert the troublesome harmonic currents in the power system. Two common types of filters are the series and the shunt filters. The series filter is characterized as a parallel resonant and blocking type which has a high impedance at its tuned frequency. The smoothing reactor used in power electronics device is an example. The shunt filter is characterized as a series resonant and trap type which has a low impedance at its tuned frequency. The single tuned LC filter is the most common design in power systems. More detailed information on harmonic filter design and applications can be found in [12,17].

Harmonic currents in a power system can also be reduced by providing a phase shift between nonlinear loads on different branches. One popular method called phase multiplication is to operate separate six-pulse static converters (12-pulse and higher) in series on the dc side and in parallel on the ac side through the phase-shifting (Δ-Δ and Δ-Y) transformers [18] so that there is self-cancellation of some harmonics. Sometimes, a specially designed transformer (zigzag) is used to trap triplen harmonic currents and to prevent the currents flowing back to the source from the nonlinear load. This zigzag transformer is usually designed to provide a low harmonic impedance between its windings compared to the source harmonic impedance. Thus there are circulating harmonic currents between the nonlinear load and the transformer.

Active filtering techniques [19] have drawn great attention in recent years. By sensing the nonlinear load harmonic voltages and/or currents, active filters use either 1) injected harmonics at 180 degrees out of phase with the load harmonics or 2) injected/absorbed current bursts to hold the voltage waveform within an acceptable tolerance. These approaches provide effective filtering of harmonics and eliminate some adverse effects of passive filters such as component aging and resonance problems.

Harmonic standards provide useful preventive solutions to harmonics. Recent standards such as IEEE 519-1992 [11] and IEC 1000-3-2 [20] emphasize placement of limits on harmonic currents produced by nonlinear loads for customers and network bus harmonic voltage distortion for electric utilities.

2.7 Summary

For harmonic studies, Fourier series and Fourier analysis are fundamental concepts. Many FFT algorithms have been implemented for DFT computations on measuring harmonics.

In nonsinusoidal situations, the conventional electric quantities used in sinusoidal environment need to be redefined. However, power definitions as well as harmonic phase sequences under unbalanced three-phase systems are still under investigation. Several harmonic indices have been defined for the evaluation of harmonic effects on power system components and communication systems.

To predict precisely the power system response to harmonics requires accurate models for power system elements and harmonic-generating loads. A simple technique for harmonic analysis is the current injection method, which is performed in the frequency domain. Other analysis methods include time domain and frequency/time domain techniques. Solutions to harmonics can be classified as remedial and preventive. Passive and active filters are widely-used remedial solutions, and harmonic standards provide the best solution before actual harmonic problems occur.

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