

Name: _____

Recitation: _____

Multivariable Calculus

Math 215 Fall 2003

Professor:

Second Exam

November 13, 2003

Please *do all your work in this booklet* and **show all the steps**.

Make sure to **box** your final answer.

You do not need to simplify your answer, but we expect you to know and simplify some basic expressions, like $\sqrt{4}$ or $\cos \pi$.
Calculators and note-cards are not allowed.

Some useful trigonometric identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

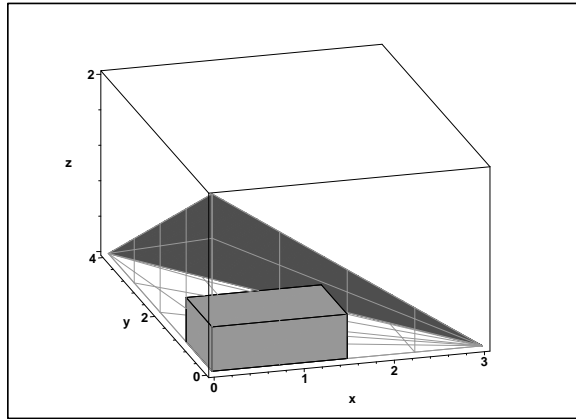
Problem	Possible points	Score
1	15	
2	10	
3	10	
4	10	
5	15	
6	15	
7	15	
8	10	
Total	100	

Problem 1. (15 pts.) Find and classify all critical points of the function

$$f(x, y) = 5x^2y - 2xy^2 + 30xy - 3$$

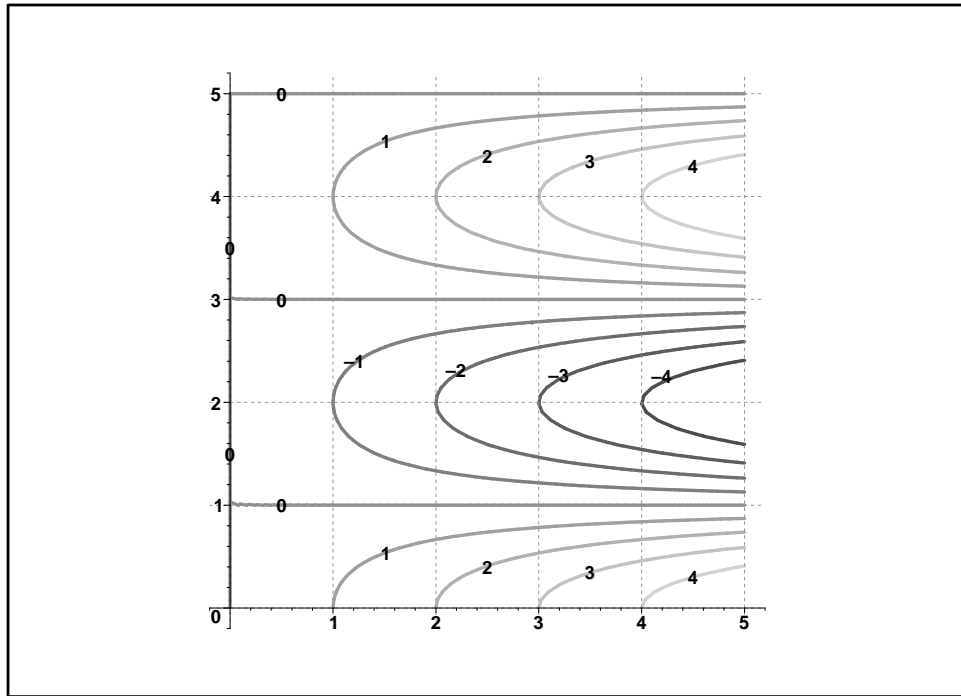
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Problem 2. (10 pts.) Use Lagrange multipliers to determine the dimensions of the storage box of the largest volume that can be pushed into the remote dusty corner of the attic as shown on the picture below.



Problem 3. (10 pts.) Consider the function $f(x, y) = x \cos\left(\frac{\pi y}{2}\right)$ over the rectangle $R = [0, 4] \times [1, 3]$.

(a-5 pts.) Using the level curves of this function on the plot below, estimate $\iint_R f(x, y) dA$ (you have to **carefully explain** your setup).



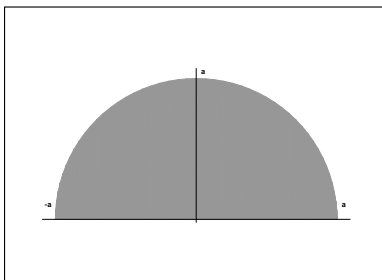
(b-5 pts.) Find the exact value of this integral.

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Problem 4. (10 pts.) Sketch the region of integration and evaluate the integral

$$\int_0^2 \int_{2y}^4 \sin x^2 dx dy$$

by changing the order of integration.

Problem 5. (15 pts.) Find the center of mass of a non-homogeneous lamina in the shape of a semi-circle of radius a cm (see the picture below) if the density function is $\sigma(x, y) = y$ g/cm².



Problem 6. (15 pts.) Consider the iterated triple integral

$$\int_0^3 \int_{\frac{2z}{3}}^2 \int_0^{4-y^2} y \, dx \, dy \, dz$$

(a-10 pts.) Sketch the region of integration E and then rewrite this integral as $\iiint_E y \, dz \, dx \, dy$, i.e., find the appropriate limits of integration (note that your sketch can be helpful).

(c-5 pts.) Evaluate this integral.

Problem 7. (15 pts.) Find the amount (mass) of ice-cream in the ice-cream cone formed by a sphere of radius 7 cm centered at the origin and a cone opening upwards from the origin with top radius of 3 cm if the density of ice-cream is given by $\sigma(x, y, z) = z \text{ g/cm}^3$ (which makes sense, since the ice-cream is more compressed near the spoon and is more loose towards the bottom of the cone).

Problem 8. (10 pts.) The graph below is a plot of some level curves for a function $f(x,y)$, along with arrows representing the gradient $\nabla(f)$ (adjacent level curves represent the same change in f). D is the region bounded by (and including) the oval constraint curve C . On the graph, please carefully mark and label the following:

- (a - 2 pts.) An example of a critical point that is a local minimum.
- (b - 2 pts.) An example of a critical point that is a local maximum.
- (c - 2 pts.) An example of a critical point that is a saddle.
- (d - 2 pts.) Location of the absolute maximum of $f(x,y)$ on the region D .
- (e - 2 pts.) Location of the absolute minimum of $f(x,y)$ on the region D .

