1. (20 Points) Below you will find a list of sentences or quotations that describe a significant figure in the history of mathematics. Identify that figure.

(a) First to use the idea of “nearer than by any given difference”, “as near as we choose”, “differ from it by as little as one wishes” in a formal proof.

(b) Arguably the greatest of all mathematicians, he discovered the Quadratic Reciprocity Theorem.

(c) Will forever be thought of as a witch in spite of a major text on calculus written in Italian.

(d) He saw it, but he didn’t believe it.

(e) Wrote the definitive work on exponential, logarithmic, and trigonometric functions.

(f) Built calculus from a harmonic triangle.

(g) A non-professional mathematician who made a significant contribution to the development of calculus through his criticism of Newton.

(h) An adequate case can be made that this man discovered calculus.

(i) He cut to the chase and gave the first formal definition of a real number.

(j) “... not before they vanish, nor afterwards, but with which they vanish ...”

2. (18 Points)

(a) use Newton’s generalization of the notation \( \binom{n}{k} \) to obtain the coefficients of \( x, -\frac{x^3}{3}, \frac{x^5}{5}, -\frac{x^7}{7} \) in the series expansion for \( \int_0^x (1 - x^2)^{\frac{8}{5}} \) \( dx \).

(b) write that same column of coefficients for \( \int_0^x (1 - x^2)^{\frac{8}{5}} \) \( dx \).

(c) write the first four non-zero terms in the series expansion of \( \int_0^x (1 - x^2)^{\frac{8}{5}} \) \( dx \).
3. (14 Points) Here are the steps in one of Cantor’s proofs that the set of real numbers cannot be listed as \( \{r_1, r_2, r_3, \ldots\} \).

(a) Choose two numbers \( a \) and \( b \) from the list in such a way that \( a < b \). Why is this possible?

(b) Continue in this way to obtain a nested sequence of intervals \( (a_n, b_n) \). What does nested mean, and why is it possible to keep obtaining the next interval in the sequence?

(c) \( \{a_n\} \) and \( \{b_n\} \) are Cauchy sequences. Why so?

(d) These two sequences are therefore convergent to two real numbers, \( a \) and \( b \) respectively. Why so?

(e) Clearly, \( a \leq b \). Why?

(f) If \( a = b \), then \( a \) is not on the original list. How do we know this?

(g) If \( a < b \), then some real number \( r \) is not on the original list. How do we know this?

4. (12 Points) Use the method of Lagrange to find the derivative of \( \frac{1}{x} \). Hint: divide 1 by \( x + i \).

5. (12 Points)

(a) Is 71 a Quadratic Residue of 73? Why or why not?

(b) Use the Quadratic Reciprocity Theorem to determine whether 3 is a quadratic residue of 79.

6. (12 Points) Use Euler’s procedure to solve the differential equations

(a) \( y'' + 6y' + 5y = 0 \).

(b) \( y''' - y'' + 4y' - 4y = 0 \).

7. (12 Points) A number \( p \) is a limit point of a set \( P \) if every neighborhood of \( p \) (open interval containing \( p \)) contains infinitely many points of \( P \). The derived set, \( P' \) of a set \( P \) is equal to the set of all limit points of \( P \). Show that \( P'' \subseteq P' \).