GENERAL RELATIVITY AT WEST POINT, 2005

CHRIS MOSELEY

1. Special Topics Course in General Relativity, West Point, Spring 2005

This story actually begins in the spring 2004 term, when I was a postdoctoral fellow at the U. S. Military Academy at West Point. I suggested to my Vector Calculus cadets that a course in general relativity based on James Hartle's recent text *Gravity* [1] would make an interesting Special Topics course. I expected perhaps one or two cadets to show interest; instead, six cadets kept emailing me the following fall, asking if the course would be offered. Although it would mean teaching an overload the following spring, I couldn't turn them down–after all, it was my idea!

I should mention here why I was surprised by the number of cadets who wanted to take this course. Cadets at West Point are preparing for a future in the Army. General relativity has little obvious relevance to an Army career, so I thought there would be correspondingly little interest. How wrong I was.

General relativity has a reputation for being difficult, perhaps because traditional treatments begin with about two months of affine connections, the Riemann curvature tensor and other machinery before students see any of the exciting topics they've heard about, such as neutron stars, black holes, and the expanding universe. The *Gravity* textbook takes the opposite approach: these topics are introduced early, with just the right amount of differential geometry needed at each step to grasp the central ideas and perform interesting calculations. This allows one to pack a good deal of geometry into the syllabus in a fairly painless way. By the end of the term, my cadets knew how to derive the geodesic equations for a given metric tensor using the Euler-Lagrange equations, how to find conservation laws using Killing vector fields, and were performing orbital mechanics calculations in Schwarzschild spacetime.

Most of the cadets had advanced placement in mathematics when they arrived at West Point, but one had started in the regular calculus sequence. He had been in a freshman calculus section I taught during my second semester at West Point, and I recognized his interest in mathematics and physics and encouraged him. He became a physics major. When the Dean's office scheduled the general relativity class, I made sure it was at a time he could attend.

When we first began discussing the concepts behind the Euler-Lagrange equations, this cadet not only understood the ideas, but was able to help me explain them to the other cadets. He had already seen the equations in his classical mechanics course, but it was still impressive to see him adapt so quickly to a more geometric context. I believe his intuition for physics problems helped him understand ideas from differential geometry.

To help all the cadets develop this kind of intuition, I assigned a second, rather unusual "textbook": Larry Niven's science fiction novel *Protector* [2], which includes a vivid depiction of interstellar spaceflight at relativistic speeds. This novel served as more than mere entertainment. The main event in *Protector* is a relativistic space "dogfight" during which two astronauts fall freely close to a neutron star as part of a gravitational perturbation maneuver. Since the neutron star in the story is described as *slowly* rotating, the environment near the star could be modeled with a Schwarzschild metric. For their group project, the cadets were required to extend their knowledge of special and general relativity to compute the acceleration, deceleration and orbital elements required to execute the perturbation maneuver while remaining consistent with Mr. Niven's story.

They did a great job, exploiting a *Mathematica* notebook provided on the *Gravity* website to numerically integrate candidate orbits in Schwarzschild spacetime near the neutron star. The orbit they eventually designed is shown in Figure 1.

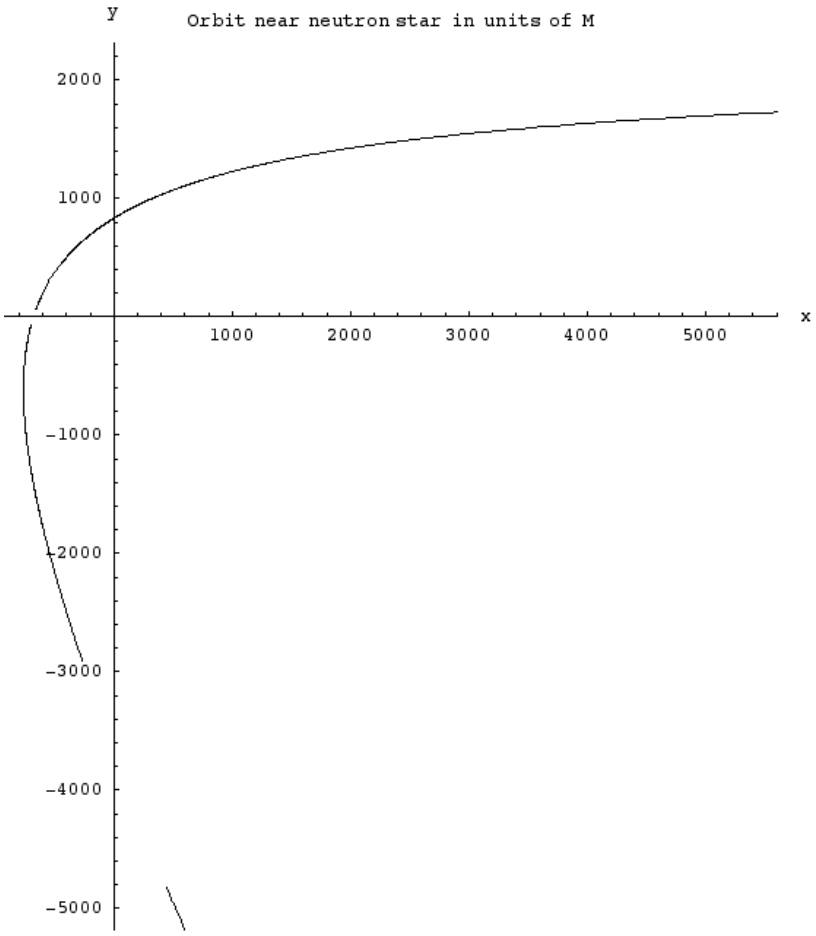

Figure 1: Escape orbit near fictional neutron star BVS-1, designed by my cadets

Their report was so good that I emailed a copy to Mr. Niven. He replied:

```
From:  Larry Niven [mailto:***********@earthlink.net]
Sent:  Monday, May 30, 2005 2:08 PM
To:  Moseley, C. DR MATH
Subject:  RE: West Point cadets' analysis of Protector

Wow.

I never dreamed anyone would actually do a relativistic analysis of that dogfight.  It's wonderful.  I
just showed it to my wife.  Then I'll watch my brother's reaction.

Thanks to you and your cadets.  I'm very pleased that they were able to make it work.

Larry Niven
```

This experience taught me never to underestimate students' interests, regardless of their intended career path. It also reminded me that students can do amazing things when they're given the opportunity, and will work very hard on assignments that are truly interesting.

Oh, and the student who arrived without advanced placement? He received a well-earned A.

References

[1] James Hartle, *Gravity: An Introduction to Einstein's General Relativity*. Addison-Wesley, San Francisco, 2003.

[2] Larry Niven, *Protector*. Ballantine Books, New York, 1973.

[3] GR course website: `http://www.calvin.edu/~cgm3/GR2005/GR.html`

Department of Mathematical and Statistics, Calvin College, Grand Rapids, MI 49546

E-mail address: `cgm3@calvin.edu`