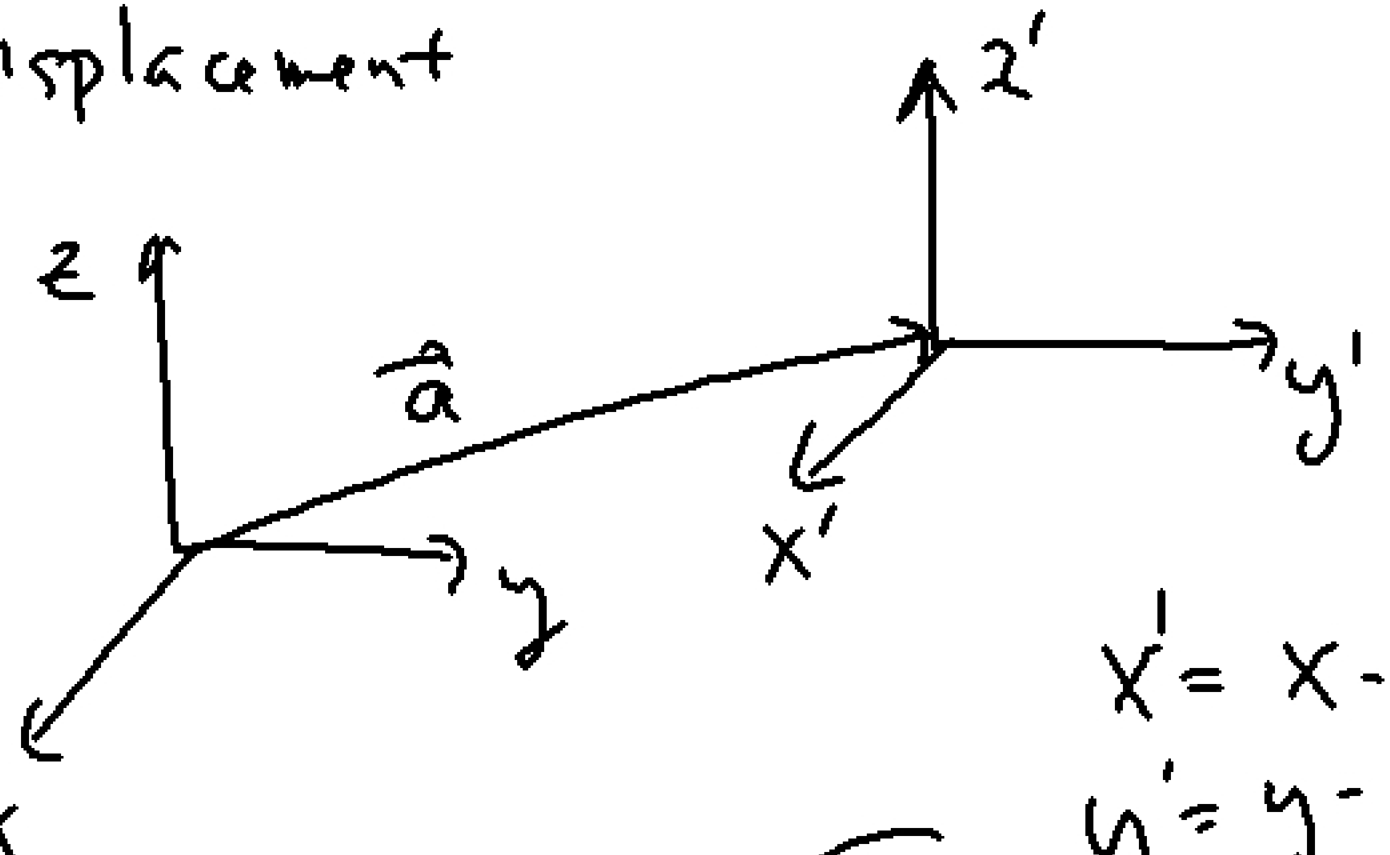
Two inertial frames can be related by: (pp 34-36)

1) displacement

$$a = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

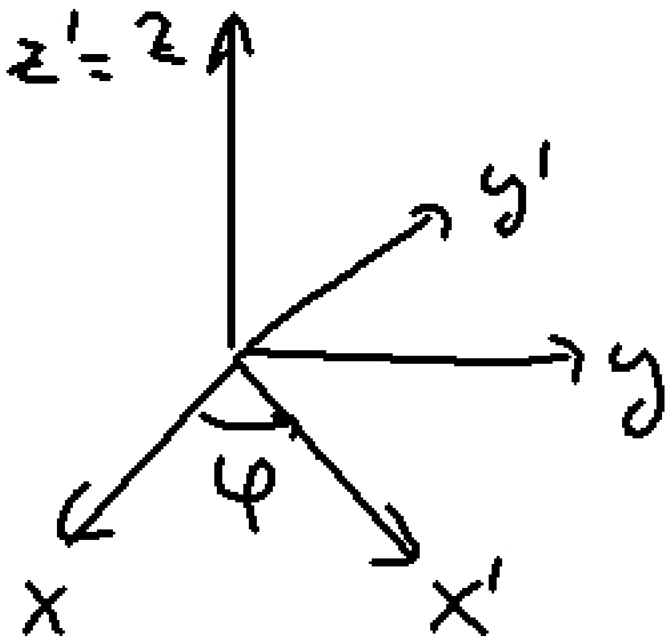


$$x' = x - a_1$$
$$y' = y - a_2$$
$$z' = z - a_3$$

$$\begin{cases} dx' = dx \\ dy' = dy \\ dz' = dz \end{cases}$$

$$ds^2 = dx^2 + dy^2 + dz^2 = dx'^2 + dy'^2 + dz'^2.$$

2) rotation



$$x' = x\cos\varphi + y\sin\varphi$$

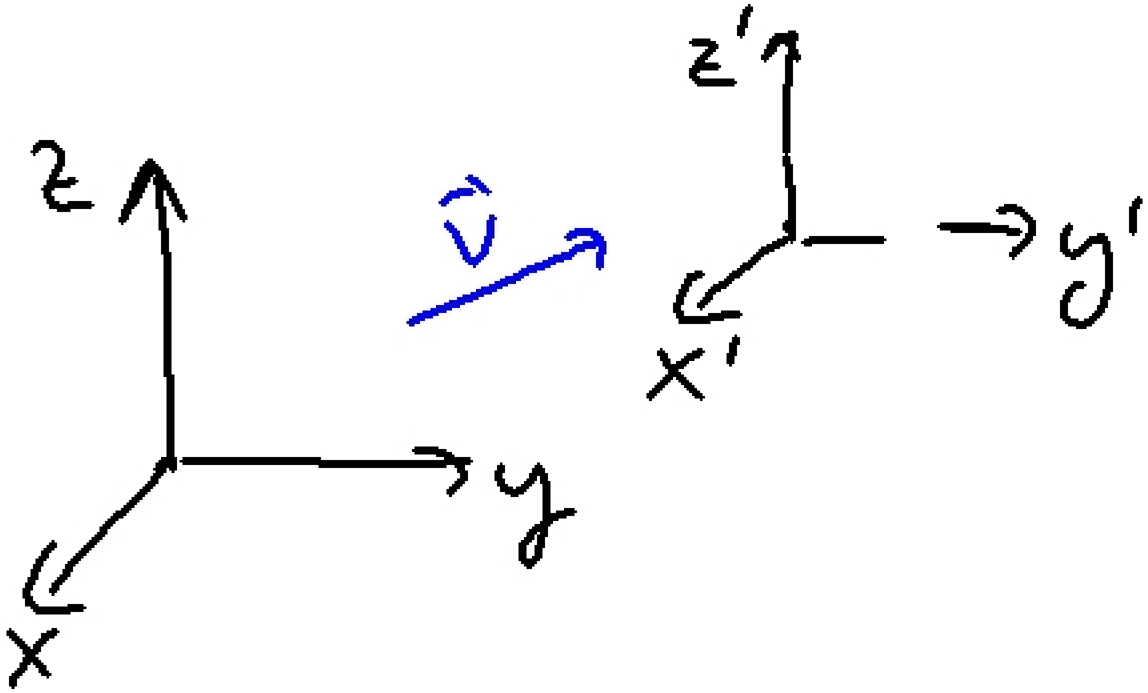
$$y' = -x\sin\varphi + y\cos\varphi$$

$$z' = z$$

(x, y, z) is inertial

$\Longleftrightarrow$ (x', y', z') is inertial.

3) uniform velocity $\vec{v}$



$\vec{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}$

$dx' = dx - v_1 dt$
$dy' = dy - v_2 dt$
$dz' = dz - v_3 dt$

$$\left.\begin{aligned} x' &= x - v_1 t \\ y' &= y - v_2 t \\ z' &= z - v_3 t. \\ t' &= t \end{aligned}\right\} \text{Galilean transformation.}$$

Any two inertial frames are related by some combination of these transformations.

1) translation (displacement)
2) rotation

Euclidean transformations

3) uniform velocity — Galilean transformation.

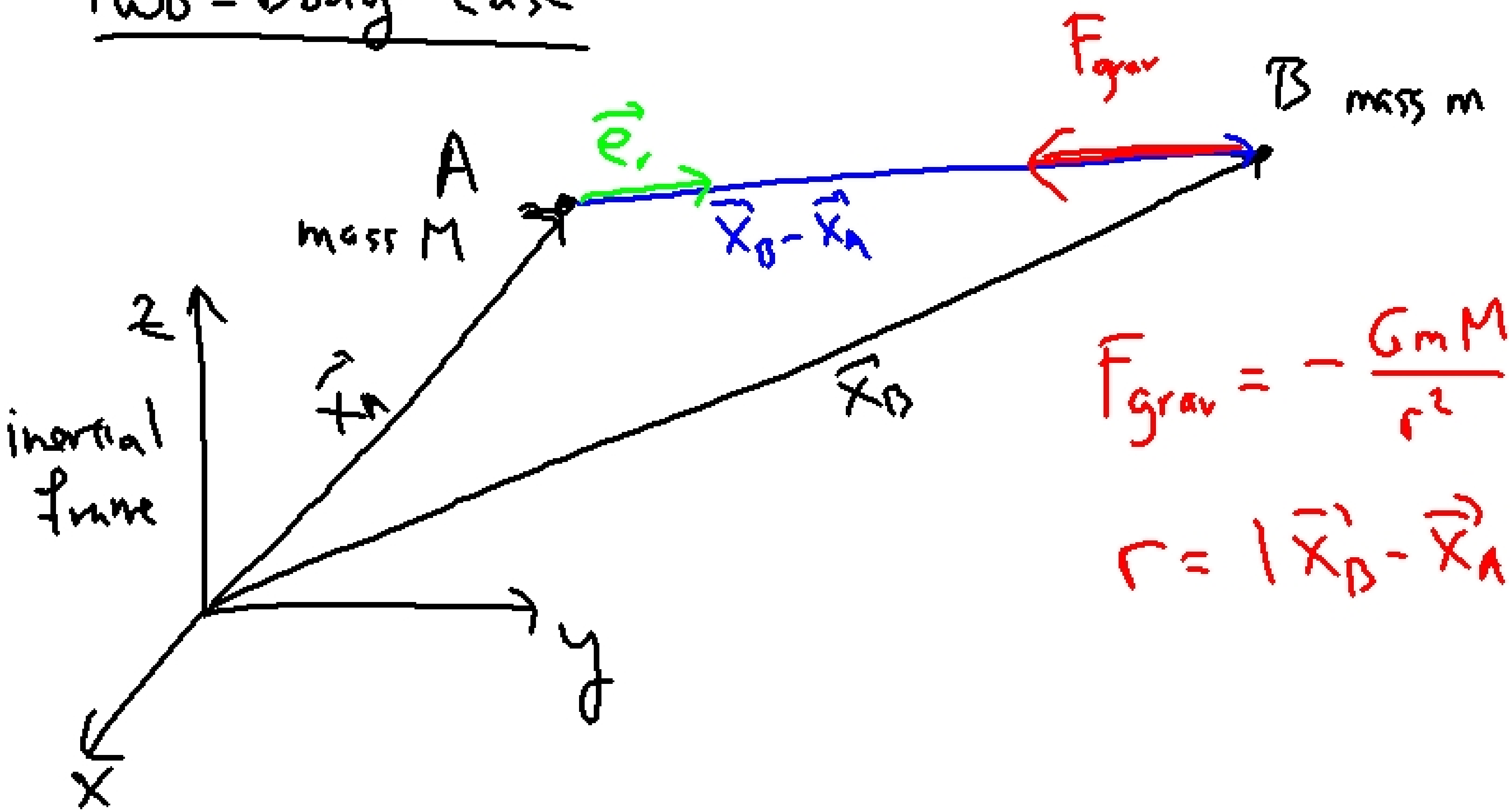
p38 does this for rotations.

Check: under a Euclidean transformation,

ds^2 remains invariant (and therefore ds).

Newtonian gravity

Two-body case

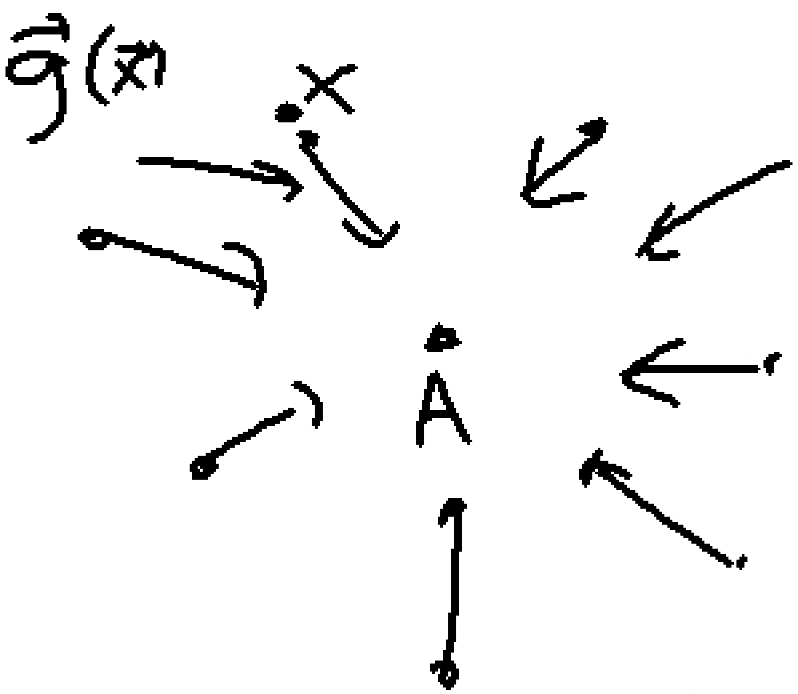


$$\vec{F}_{grav} = -\frac{GmM}{r^2}\vec{e}_r$$

$$r = |\vec{x}_B - \vec{x}_A|$$

$$F_{grav} = -\frac{GmM}{r^2}\vec{e}_r = m\left(-\frac{GM}{r^2}\vec{e}_r\right)$$
$$= m\vec{g}$$

where $\vec{g} = \frac{-GM}{r^2}\vec{e}_r$ is the gravitational vector field



vector field $\vec{g}$

$\vec{g}$ is defined on $\mathbb{R}^3 - \{x_A\}$. ← simply connected region.

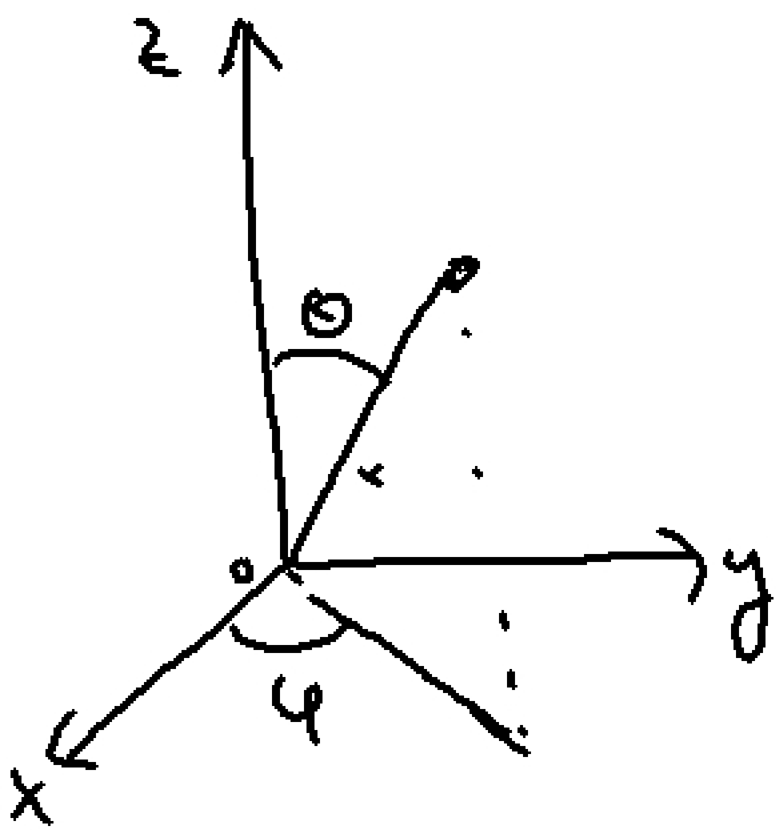
A word from our sponsor, $\vec{\nabla}$.

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \quad \text{(rectangular coordinates)}$$

$$\vec{\nabla} \times \vec{g} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ g_x & g_y & g_z \end{vmatrix}$$

In rectangular coordinates.

Assume (without loss of generality) that A is at the origin.



$\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi$: unit orthogonal vectors for spherical coordinates

$$\vec{g} = -\frac{GM}{r^2}\vec{e}_r$$

In spherical coordinates

$$\vec{g} = g_r \vec{e}_r + g_\theta \vec{e}_\theta + g_\varphi \vec{e}_\varphi = -\frac{GM}{r^2}\vec{e}_r$$

$$\Rightarrow g_r = -\frac{GM}{r^2},\ g_\theta = 0,\ g_\varphi = 0.$$

Div Grad Curl, 3rd ed, p. 85:

$$(\vec{\nabla}\times\vec{g})_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, g_\theta) - \frac{1}{r\sin\theta}\frac{\partial g_\theta}{\partial\varphi} = 0$$

$$(\vec{\nabla}\times\vec{g})_\theta = \frac{1}{r\sin\theta}\frac{\partial g_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(r g_\varphi) = 0$$

$$(\vec{\nabla}\times\vec{g})_\varphi = \frac{1}{r}\frac{\partial}{\partial r}(r g_\theta) - \frac{1}{r}\frac{\partial g_r}{\partial \theta} = 0$$

$\Longrightarrow$ $\vec{\nabla}\times\vec{g} = 0$ at every pt $\vec{x} \neq 0$.

in a simply-connected region

$\Longrightarrow$ $\vec{g} = -\nabla\Phi$, where Φ is a potential function (defined up to additive constant)

$$\boxed{\Phi = -\frac{GM}{r}}$$