<u>This week</u>: scream through ch 3 (Newtonian gravity)

Sect 3.5 is the most important (variational calculus).

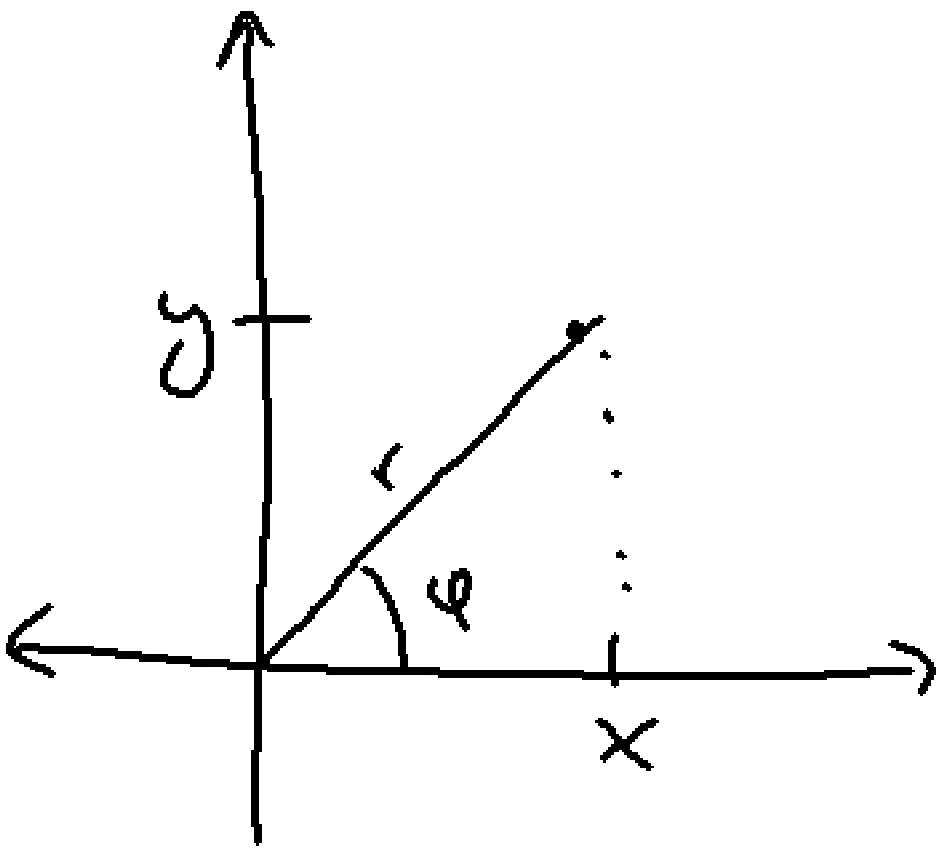
Last time: dS is an <u>invariant</u>, i.e. independent of coordinate system used.

Example: flat plane

$$dS^2 = dx^2 + dy^2 \quad \text{(rect coord)}$$
$$= dr^2 + (r\,d\varphi)^2 \quad \text{(polar coord)}$$

$$dx = dr\cos\varphi + r\,d(\cos\varphi)$$
$$= dr\cos\varphi - r\sin\varphi\, d\varphi.$$

Proof that these are equal:



$x = r\cos\varphi$

$y = r\sin\varphi$

$\boxed{\text{chain Rule for differentials:}}$

$$dy = dr\sin\varphi + r\cos\varphi\, d\varphi$$

$$(dx)^2 = (dr\cos\varphi - r\sin\varphi\, d\varphi)^2$$

$$= (dr)^2\cos^2\varphi - 2r\sin\varphi\cos\varphi\, dr\, d\varphi + r^2\sin^2\varphi\,(d\varphi)^2$$

$$(dy)^2 = (dr)^2\sin^2\varphi + 2r\sin\varphi\cos\varphi\, dr\, d\varphi + r^2\cos^2\varphi\,(d\varphi)^2$$

$$(dx)^2 + (dy)^2 = (dr^2)(\cos^2\varphi + \sin^2\varphi) + r^2(\sin^2\varphi + \cos^2\varphi)(d\varphi)^2$$

$$= (dr)^2 + (r\, d\varphi)^2.$$

$\therefore$ these two expressions are equal, as they should be!

ch 3 Newtonian Physics

Reference frame (frame): coord. system, sometimes based in a lab

In general, a frame only covers a limited region of space and time.

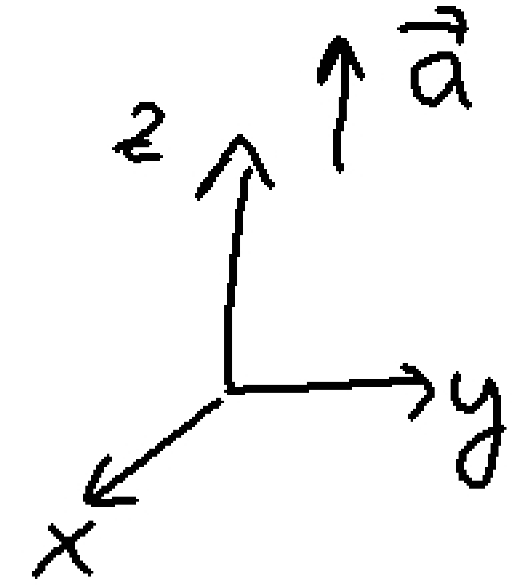
Newtonian mechanics
Special relativity } exceptions: a single frame suffices.

Inertial frame : one in which Newton's Laws hold good.

More specifically: if a free particle (no net forces acting on it) is moving with coordinates $(x(t), y(t), z(t))$ in an inertial frame, then

$$\frac{d^2x}{dt^2} = 0, \quad \frac{d^2y}{dt^2} = 0, \quad \frac{d^2z}{dt^2} = 0.$$

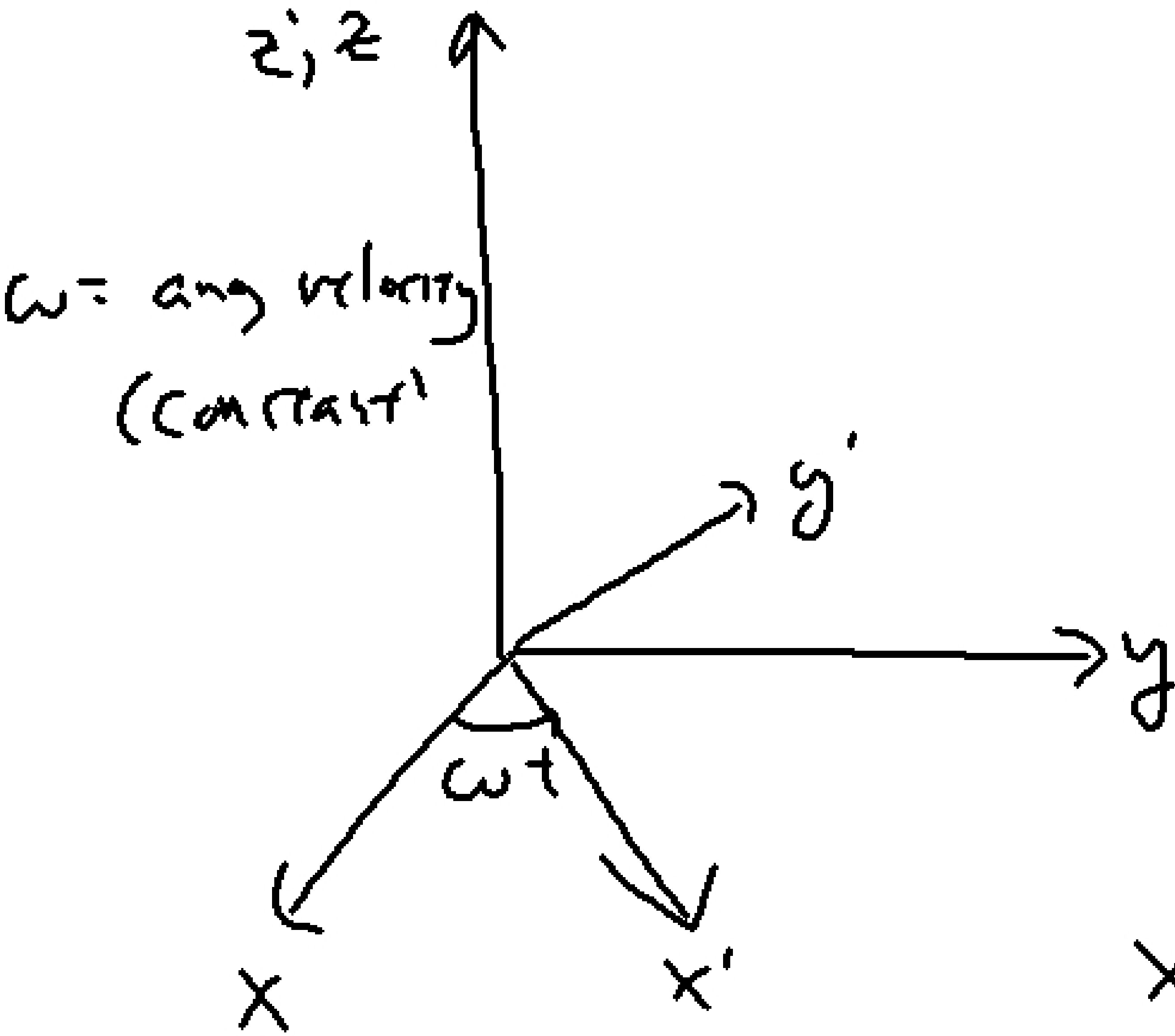
Non-inertial frame examples:

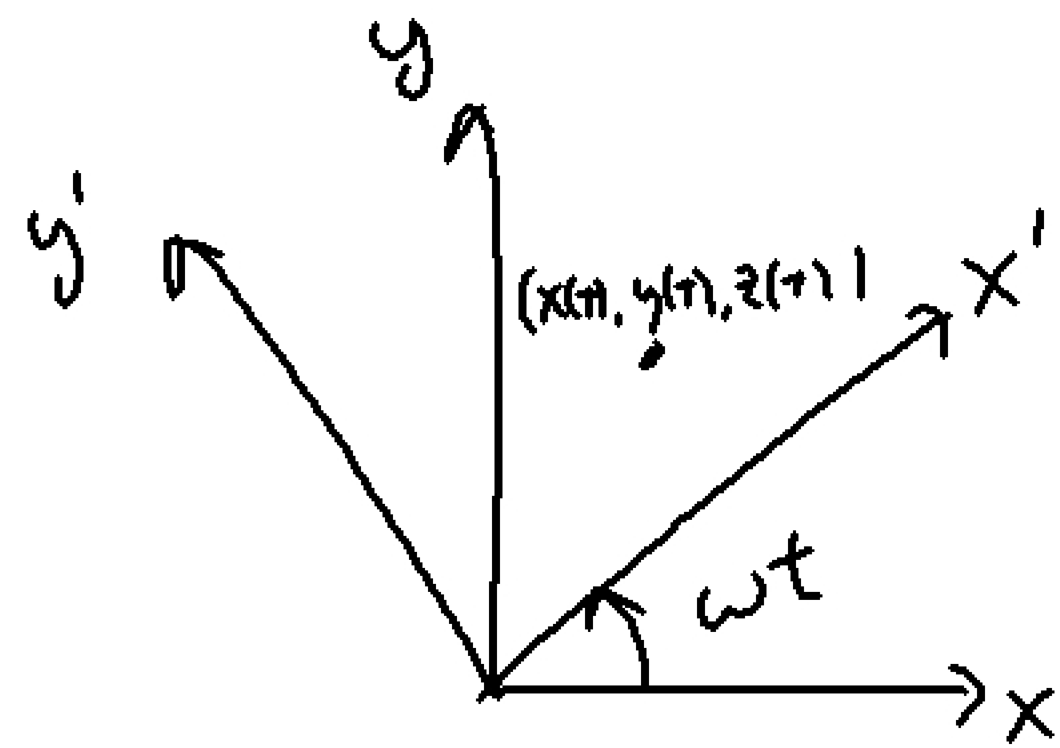


1) accelerating space shuttle

2) frame rotating at constant ang. velocity ω with respect to an inertial frame.

Example: (x, y, z) inertial frame
(x', y', z') rotating frame





$$\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0.$$

$$x'(t) = x(t)\cos\omega t + y(t)\sin\omega t$$
$$y'(t) = -x(t)\sin\omega t + y(t)\cos\omega t$$
$$z'(t) = z(t).$$

$$x'(t) = x(t)\cos\omega t + y(t)\sin\omega t$$

$$\frac{dx'}{d\tau} = \frac{dx}{d\tau}\cos\omega t - x(t)\,\omega\sin\omega t + \frac{dy}{d\tau}\sin\omega t + y(t)\,\omega\cos\omega t$$

$$\frac{d^2x'}{d\tau^2} = \cancelto{0}{\frac{d^2x}{d\tau^2}}\cos\omega t - \frac{dx}{d\tau}\omega\sin\omega t - \frac{dx}{d\tau}\omega\sin\omega t - x(t)\omega^2\cos\omega t$$

$$+ \cancelto{0}{\frac{d^2y}{d\tau^2}}\sin\omega t + \frac{dy}{d\tau}\omega\cos\omega t + \frac{dy}{dt}\omega\cos\omega t - y(t)\omega^2\sin\omega t$$

$$= -2\frac{dx}{dt}\omega\sin\omega t + 2\frac{dy}{dt}\omega\cos\omega t - \omega^2\left(x(t)\cos\omega t + y(t)\sin\omega t\right)$$

$$= 2\omega\left(-\frac{dx}{d\tau}\sin\omega t + \frac{dy}{d\tau}\cos\omega t\right) - \omega^2 x'(t).$$

If in addition we assume that $(x(t), y(t), z(t))$ is constant

(particle motionless with respect to inertial frame)

$$\left.\begin{aligned} \frac{d^2 x'}{dt^2} &= -\omega^2 x'(t) \\ &\text{and likewise} \\ \frac{d^2 y'}{dt^2} &= -\omega^2 y'(t) \end{aligned}\right\}$$

appears to have centripetal acceleration in x', y', z'.