Moreno-Socías Conjecture in 3-Variables

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When working with algebra, roots of the polynomials are frequently sought for. An ideal is a tool that help us compute its roots by replacing complicated polynomials with simpler polynomials. Moreno-Socías conjectured that for every ideal, the initial ideal is a weakly reversed lexicographic ideal when ordered using graded reverse lexicographic ordering. The proof of the conjecture was given by Moreno-Socías in his paper. However, we will be proving the conjecture with a different method by producing an explicit basis. The 2-variables case have been proved in another paper. In this research, we focus on proving Moreno-Socías conjecture in 3-variables case.

Initially, we studied *Ideals, Varieties, and Algorithms* by Cox, Little, and O'Shea to better understand the conjecture. Afterwards, we studied the proof for the 2-variables case of the conjecture, written by Aguirre, Jarrah, Laubenbacher, et al. to examine the method. We then defined the polynomials for 3-variables case and generated the Gröbner basis from the polynomials. We used computer calculations in *Maple* and, as calculations became protracted, used *Sage* and *Singular* to generate the Gröbner basis. We then endeavored to find the patterns in the leading monomials of the minimal generating polynomials Gröbner basis.

We refined the procedure of generating the Gröbner basis by renaming the coefficients for every basis generated. As the polynomials in study are algebraically independent, the replacement of coefficients are without loss of generality. Any coefficient that became 0 are produced from formal cancellations. Apart from that, we made a program using *Sage* and *Singular* to aid us in generating the Gröbner basis. Alongside that, we have found the general case for the first three polynomials of the Gröbner basis. We found the minimal Gröbner basis for several specific cases, but have found no general case as of yet though patterns have emerged that we are seeking to exploit.

Aside from giving me a real experience in pure mathematics research, my involvement in this research has shown me the boundless field of mathematics. Despite the rapid progress in science, I have come to realize the expanse of undiscovered knowledge that humans might still be able to utilize. This research has also taught me the importance of perseverance and creativity in handling perplexing results in hopes of justifying the results logically.