



PROJECT PROPOSAL AND FEASIBILITY STUDY

BOAT!

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DECEMBER 9, 2005



Abstract

This report contains information pertaining to the fall 2005 portion of the senior design project at Calvin College for team 9: BOAT!. It includes a project proposal with an emphasis on the feasibility of the project. The goal is to design and assemble a human-powered watercraft which utilizes a hydrofoil design to minimize drag, increase speed, and make the craft an enjoyable alternative to the conventional paddleboat. The report includes alternatives considered, calculations performed, and selections made over the course of the semester.

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1 Introduction

The intention of BOAT! is to research, design, and construct a human powered boat that is an alternative to the traditional paddle boat. This craft should be capable of higher speeds and more maneuverable than a paddleboat. The overall goal is to produce a watercraft that provides the user with more entertainment.

Because added weight will reduce the maximum speed, the watercraft will only be suitable for one user. Also, drag plays a large role in determining speed. To deal with this, a hydrofoil design will be implemented. Hydrofoils act like an airplane wing under water, which creates lift and raises the watercraft out of the water; in other words the watercraft is “flying” in water. With the hull out of the water, drag on the hull is greatly reduced, and speed increases.

Another primary objective is to create a watercraft with more responsive handling than the ineffective joystick and rudder system of the typical paddleboat. The response of the handling is important since the aim is to achieve higher speeds.

The research, design, and construction done by BOAT! is to complete requirements for Engineering 339/340 at Calvin College. This course serves as the capstone course necessary for graduation from the Engineering program. The focus for the course is a year long project that is initiated and brought to completion by teams of four to five students. BOAT! team members are each mechanical engineering students and include: Luke Joyce, Marcus Krosschell, Marcus Pettinga, and Zac Snyder.

2 Design Norms

A design should not be influenced exclusively by technical attributes. One needs to consider design norms and how their design will affect others. BOAT! has considered the design norms of cultural appropriateness, stewardship, and transparency in the design of this watercraft.

A good design fits well into the culture it is introduced to. In the United States, many Summer time activities revolve around bodies of water. Recreational boating is nothing new to the American public. This design also helps the health conscious as it would provide a good work out for the rider while still feeling like a leisurely activity.

This craft also fits into the design norm of stewardship. Because the craft is powered entirely by the rider, it has no environment harming emissions. God has created this world for his people to enjoy. This watercraft allows the rider to enjoy that creation while still maintaining a healthy environment for other creatures to live in. Because the ride is quiet, the rider can observe other animals without disturbing them. As previously stated, riding this watercraft promotes good health. It allows the user to be a good steward of their earthly bodies as well as the environment.

The design should be transparent so that with only brief instruction anyone can effectively use the watercraft for their own entertainment. The joy of using the final product should not be dependant on one's ability to know any advanced instructions or have any uncommon abilities. It should be no more difficult to use than a bicycle.

3 Preliminary Feasibility Analysis

3.1 Similar Products

An important part of the design process is to learn what similar products or techniques are currently on the market. A team from MIT worked on a similar design from 1989 through 1993, named The Decavitator. They were awarded the DuPont prize as the fastest recorded speed for a human powered watercraft as of December 31, 1992. Their world record run was recorded at 21.3 miles per hour. Because the speed requirements are more modest for this application, the Decavitator's success suggests that this project is feasible.

The Flyak is another design that suggests the feasibility of this watercraft. The Flyak is a kayak fitted with hydrofoils. It demonstrates how easy attaining flight speeds can be as its rider is more than capable of reaching those speeds with a typical kayak paddle and a reasonable amount of effort. The FlyakOne, designed by Einar Rasmussen, currently tops out at 16.9 miles per hour.

3.2 Hydrofoil Design and Construction

Early on in the project the hydrofoil was recognized as the most complex component of the design. Because of this the feasibility of the entire project hinges on this one component. Much research has been done to determine the most effective method to ensure successful hydrofoil design and implementation. Through this research many different hydrofoil profiles and their applications have been studied to determine what would work best for this watercraft. This research has proven fruitful as concerns over feasibility have been relieved. Similar profiles have been used for similar projects in the past. Research also turned up useful information on hydrofoil construction techniques to prove that with a reasonable amount of effort the design and construction of a hydrofoil will not be a stumbling block.

3.3 Material Availability and Budget Considerations

As we began to explore our various design options, we found many items for which specialized materials would be ideal, such as in the forming of the hydrofoils and struts, the gear reduction in the drive train, the driveshaft, the hull, framing, and a propeller, among other items. Comparing these needs to our supplied budget, we initially had concerns for feasibility. However, we have resolved this problem with a number of solutions.

Firstly, after talking to Calvin College metal shop manager Dave Ryskamp, problems pertaining to material availability on the basis of budget were relieved. Some metal can be procured from Calvin's metal shop supplies at no cost, freeing money to be spent on other necessary materials for the project.

Secondly, we were able to salvage several used items for use in our design, such as bicycle sprockets for gear reduction, a windsurfer for a hull, a gear box from an old outboard motor for power translation, and a propeller already produced for human-powered boat applications. All these items were provided to us at little or no cost, allowing us to devote the majority of the budget to more complicated components which require customized construction. A summary of our project budget is presented in Appendix A.

3.4 Time Limitations and Scope

This project has a relatively large scope with respect to its time frame. Fortunately, this was recognized early in the planning stages. The project was broken down into components, and assigned leaders to each component. While all group members are involved in the design of each component, each person's main

focus was devoted to one component. Strict time limits were also set for the design portion of the project, knowing that success will be dependent on testing and making appropriate modifications. Sufficient testing will require quite a bit of time, and by planning the timeline from an end result and working backwards, it is assured that the project scope will fit into the required project timeline.

4 Project Breakdown and Management

A human powered hydrofoil boat contains many components. To better organize our research and design we have identified and separated the boat into four main components: hydrofoil, body, steering, and propulsion. Some of these main components feature important sub-components. The body is broken up into hull and frame and the propulsion is divided into propeller and drive train.

All four main components are too complex for one person to be responsible for on their own. Therefore, each component is analyzed by all four members with one member serving as a component leader. The component leader's responsibility is to delegate tasks, lead meetings, and ensure that all phases of component work are done well and on schedule. It is also their responsibility to recognize and point out where their component overlaps with another during design of other components.

Because everyone is involved with each component, one component is focused on at a time. A period of time proportional to the amount of work required is designated for work on each particular component. When that component's design is completed, work on the next component begins under the direction of the next component leader.

5 Hydrofoil

5.1 General Requirements

The purpose of the hydrofoil is to provide lift like an airplane as well as some control and stability. As lift is created, the watercraft will rise from the surface of the water and “fly” on only the two hydrofoils. This reduces drag and speed can be increased. The implications of several designs are discussed in this section.

5.2 Configuration

5.2.1 Alternatives

Figure 1 shows the two options considered for the configuration of the hydrofoils on this craft. The first option is a V foil. Under this configuration the foil is in the shape of a V with the outer edges piercing the surface. A V foil automatically adjusts the height of the craft above the surface of the water by finding equilibrium with the area submerged and the lift force. A V foil is generally less stable due to the direction of the imposed lift force. It also increases drag as it is surface piercing and creates a turbulent flow at its edges. However, it is simpler and involves fewer moving parts due to the fact that no pitch or height control is necessary.

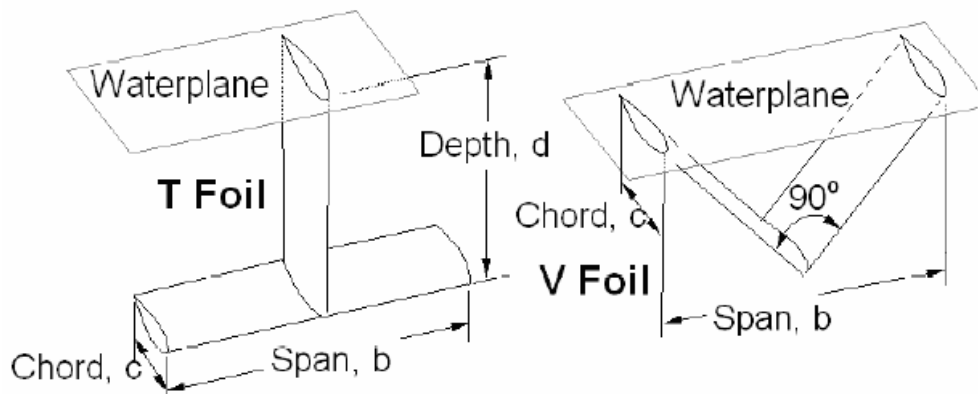


Figure 1: Possible Hydrofoil Configurations

The second option is the T foil configuration. With this option the foils are flat in the water and remain fully submerged. The T foil is easier to fabricate as there is no bend in the middle. It also provides less drag and more lift with the same plan form area. One reason that it produces less drag is because it can be supported by a single strut connection. Every part of the boat that is in contact with the surface of the water produces a drag force called spray drag. Because the T foil only has one interaction with the water it has significantly less spray drag. Another reason that it can produce greater lift and less drag than a V foil configuration is that the lift force is in the vertical direction only. A V foil configuration has horizontal lift forces that cancel each other out, so a portion of the foil is not being used to generate upwards lift, hence the total area must be increased. However, the T foil controls may need to be more complicated than the V foil because it does not automatically adjust to height and roll as the V foil does.

5.2.2 Selection

The T foil configuration has been selected for this project for a number of reasons:

- V foil design has greater drag and will produce ventilation. Ventilation is when air gets sucked down below the surface of the water and results in higher drag.
- The T foil has fewer specialty materials needed for the fabrication process. It can be made from a single piece of foam, wood, or other internal material.
- T foil produces a higher lift versus drag coefficient within a given velocity range.

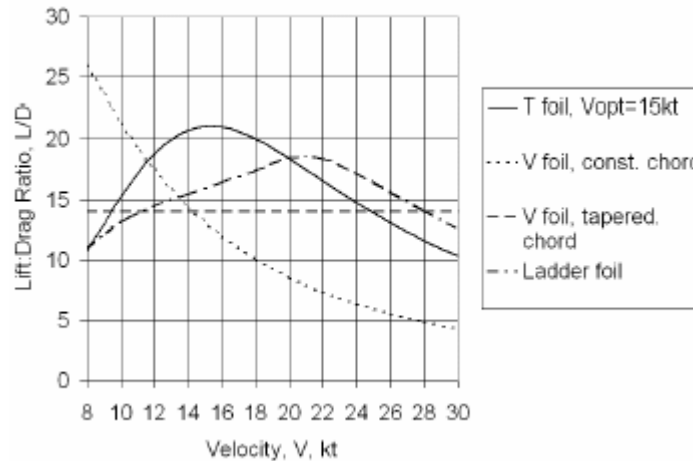


Figure 2: Lift/Drag vs. Velocity of Various Foil Configurations

- Stability control from the rear foil is no longer an issue due to our design of the automatic control system located in the front foil, therefore the benefits of a V foils stability are not needed.
- In order to reduce drag the power transmission from the pedals to the prop will go through the strut into the foil. A T foil has a strut inline with the prop. The V foil's struts are on either side and extra equipment would be needed to transmit power through the off centered strut.

5.3 Size and Shape

5.3.1 Alternatives

The size and shape of the hydrofoil is unique to this design. The slightest change in either of these variables can result in drastic changes on the performance of the craft. The determination of shape and size are based on many variables. Several of the key variables are:

- Velocity
- Structural strength
- Fluid properties
- Surface finish
- Desired lift and drag coefficients

Hydrofoils have many different cross sectional shapes. Each shape is documented as a set of coordinates in airfoil databases. The cross sectional shapes of hydrofoils are similar to the shapes of airfoils. Two

shapes were chosen for analysis based on comments from previous hydrofoil designers and their boat designs. Pictured in figures three and four are the two profiles evaluated for this boat: the Eppler E817 and the Speer H105.

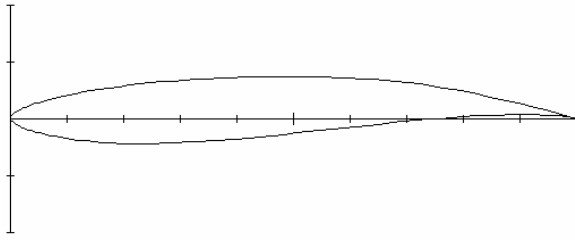


Figure 3: Eppler E817 Hydrofoil Profile

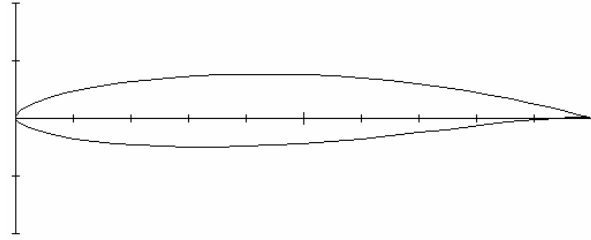


Figure 4: Speer H105 Hydrofoil Profile

Each of these profiles was designed to be used as hydrofoils. They differ from foil profiles used for flight in air in that they minimize cavitation. Cavitation is a phenomenon that occurs when the magnitude of the pressure differential between the top and the bottom of the foil is large enough that the negative pressure along the top of the foil drops below the vaporization pressure of water. When this happens the water boils and turns into a vapor. This vapor moves around the foil to the back end which results in a force in the backward direction. This causes the boat to stall and fall down to the surface. Even the slightest amount of cavitation can cause drag by forcing the liquid water flow to become severely turbulent which increases drag. Each of the above profiles is designed to avoid that effect.

5.3.2 Selection

The first step in evaluating the size of the foils was to relate many of the variables to the maximum tensile stress of the foils, sigma. To accomplish this, the coordinates of the foil were put into an excel document and then through a process of trapezoidal approximation and other integration. The moment of inertia for the foil was based on 1/8 inch thick fiberglass walls and no internal structure support. Treating the foil as a cantilever beam with a distributive load equal to the weight of the craft and combining the equations shown below with several other equations that relate velocity to lifting force.

$$\sigma = \frac{Mc}{I}, \quad M = \frac{L^2}{2} \cdot \frac{F_{lifting}}{2L}$$

This yields a relationship between lifting force, length, chord, sigma, and velocity. An intricate EES document was created that combines and solves for many different variables based on only a few values given by the user. This document produced the following characteristics of length and chord (the distance from the front to the back of the profile):

- Length of rear foil = 2.057 meters
- Chord of rear foil = 0.2411 meters
- Length of front foils = 0.330 meters
- Chord of front foils = 0.25 meters

The details of these calculations are available in Appendix B and C.

After careful analysis, the Speer H105 foil was selected for this craft. The reasons for this selection are as follows:

- The thickness of the H105 foil is 12.5% of the chord length. The E817 foil's thickness is 10.9% of the chord. The added thickness without added drag increases the structural strength.
- The H105 has a more even pressure distribution along the chord of the foil than the E817 which is preferred for optimal performance.
- The H105 is able to attain a higher coefficient of lift before cavitating at low speeds.

The details of all the hydrofoil calculations are available in Appendix B and C.

5.4 Placement

5.4.1 Alternatives

Many different arrangements of hydrofoils exist. In general, most hydrofoil crafts have two foils, one in front of the other. The location and size of these foils varies between the different crafts. Each option depends completely on the design of the craft and its intended use. Figure 5 displays a few of these options.

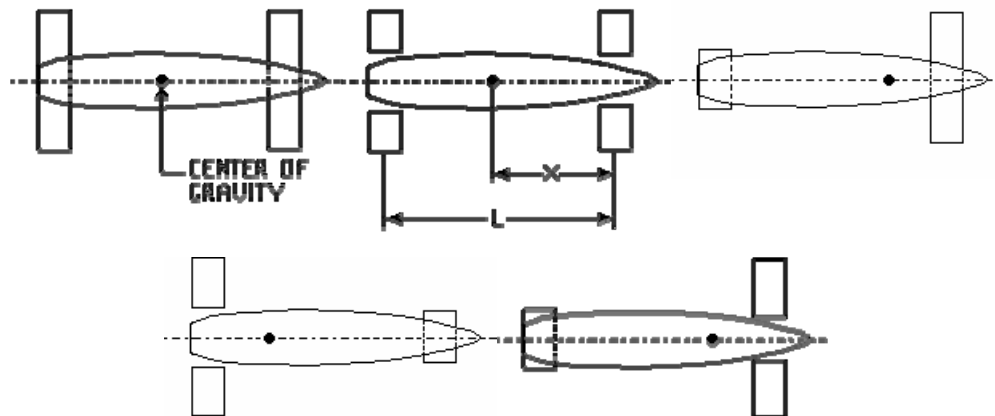


Figure 5: Possible Hydrofoil Placements

5.4.2 Selection

The foils will be placed in canard formation, i.e. a large rear foil will produce the majority of the lift needed, roughly 85%, and a smaller front foil will maintain depth, steer, and provide stability control. The front foil was split up into two separate foils in order to control roll. For further information and explanation of control see section 5.5. After finding the center of gravity of the craft the location of the front and rear foils was determined based on many other variables. The distances of the foils from the back edge of the windsurfer are:

- Rear foil = 1.277 meters
- Center of gravity = 1.788 meters
- Front foils = 3.353 meters separated 1 meter and centered over the board

5.5 Control

5.5.1 Stability

5.5.1.1 Alternatives

Lateral stability of a hydrofoil craft can be maintained through several different avenues. The first way it can be maintained is through the use of a V foil. V foils are designed such that the lift they produce varies on depth of each side. As depth is increased, more of the foil is in contact with the water which then produces more lift. Similarly, if a boat using a V foil tips to the side, the foil is raised out of the water on the opposing side and submerged further on the tipping side. This produces a moment because of unequal forces and rights the boat back so each side of the V foil has equal its force with the opposite side. This configuration is beneficial in that it is simple and requires no moving parts. Also, the V foil can be constructed in such a way that it achieves a specific desired depth at certain speeds. This is because the V foil is only partially submerged. As it rises through the water it becomes less submerged, thus producing less lift. By balancing the lift forces between the front and back hydrofoil (i.e., adjusting the design of each hydrofoil), the pitch of the craft can be controlled and a level trajectory can be maintained.

The next way to control lateral stability is with the implementation of a dihedral angle. The dihedral angle is the angle between both sides of a T foil when looking through the plane perpendicular to the axis parallel to the board. See Figure 6 below for an example of a dihedral angle on an airplane.

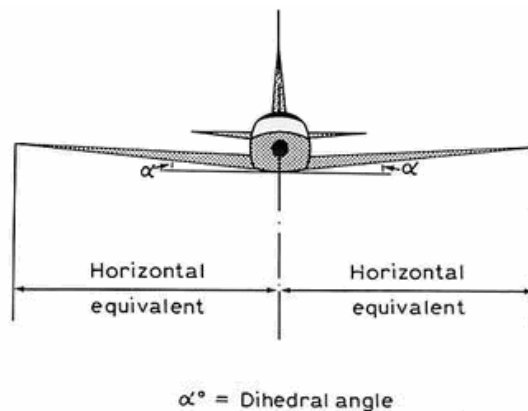


Figure 6: Airplane Dihedral Angle

Depending on where the center of roll for the craft is, a slight roll in either direction would theoretically change the forces of each side of the foil and cause slipping from side to side. The foils would in effect slide themselves back underneath the center of gravity.

The last approach to stability is to have two foils in the front of the boat which change their angle of attack and thus their lifting forces depending on their depth. This can be achieved by using two devices known as feeler arms, one on either side of the craft connected to its own hydrofoil with a means to pivot up and down about the strut. This device can be seen in figure 7.

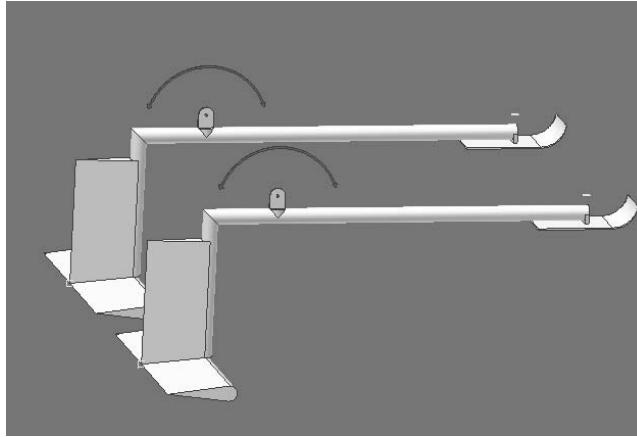


Figure 7: Proposed Feeler Arm Device

Each foil would have a surface sensing mechanism that skims across the surface of the water like a water ski. As the boat rolls to either side it would increase or decrease the depth of either side of the craft depending on which way it is rolling. The change in depth would pivot the feeler arms, changing the angle of attack of the hydrofoils, balancing the lift forces, and correcting the roll of the craft. Another advantage of this mechanism is the automatic adjustment of pitch. This happens as both feeler arms move up and down together, automatically finding the proper depth of the front foils. This depth can be adjusted by changing the angle at which the hydrofoil is connected to the feeler arm.

5.5.1.2 Selection

Based on the analysis for this boat the best alternative approach for lateral stability was the dual front hydrofoils. The V foil was already ruled out because of issues with drag, material strength, and fabrication processes outlined in section 5.2. The use of a dihedral angle on the T foil is adequate for maintaining stability in theory. In practice the difference between the resetting moment, the moment caused by the foils to counteract the tipping moment, and the moment caused by a shift in the center of gravity, is so small that almost any outside interaction would result in a crash. A dihedral would depend largely on the operator balancing their body's weight over the foil which would be difficult when pedaling at high speeds. The surface sensing foils, although they bring added complexity to the design, are the most reliable approach for the stability of this craft.

5.5.2 Steering

5.5.2.1 Alternatives

The ability of the rider to steer the craft is important, as we do not want to limit the rider to a straight line path. To control the steering of the craft, there are several options. While considering these options, we want to keep the steering mechanism simple enough that it does not add unnecessary weight or drag to the hydrofoil.

One option is to simply add a rudder. This would mean adding a separate mechanism to control the rudder. This option would add drag as we drop another component into the water.

Another option is to put flaps on the hydrofoils, like ailerons on an airplane wing, so that it turns like an airplane. This involves a more sophisticated mechanism to control the steering, but would probably add little extra weight or drag.

One more option is to allow the front hydrofoils to rotate left and right, connecting them to the body with rudder-shaped struts. Since the strut design will already incorporate a foil-shaped profile, this would involve little additional work. We would need only to transmit the rotation back to the operator with a fairly simple mechanism.

5.5.2.2 Selection

We have chosen to allow the front hydrofoils to rotate left and right, resulting in a turning motion due to the rudder-shaped struts. A simple linkage will connect the struts to operator-controlled handles, designed to pivot about the seat support, being held down to the sides of the operator. In the interest of minimizing weight, this linkage will be constructed using tightly drawn wire, rather than weak plastic rods, bulky metal rods, or expensive carbon fiber rods.

5.6 Materials

5.6.1 Alternatives

The design of the foil varies greatly with the choice of materials for construction. For the inner core the material choices are; wood sanded to profile, Styrofoam sanded to profile, aluminum skeleton, or a combination of these three. The shape of the outer shell of the foil is crucial to its performance. Any slight imperfection will be a catalyst to turbulent flow over the entire foil. The materials for the outer core are also the most expensive materials used for the boat. The outer shell material choices are: S-fiberglass with epoxy, carbon fiber, Kevlar, aluminum sheeting, and paint on wood.

5.6.2 Selection

A cost analysis on the foil produced a minimum cost using foam and an aluminum skeleton for the inner core and several layers of fiberglass for the outer shell. The aluminum skeleton adds extra structural strength to the core of the foil which cuts down on out material cost for the fiberglass shell. It also allows for a reliable connection to the strut which connects the foil to the boat. The last layer of fiberglass will be sanded down to the appropriate shape minus any imperfections and then painted with a coat of primer followed by a coat of marine boat paint for the final surface finish.

6 Body

6.1 General Requirements

The body of this watercraft must be capable of providing enough buoyancy to adequately support a rider of up to 250 pounds and additional weight from the other components. A good design would be lightweight and provide stability during foilborne operation. Specification considerations include drag, weight, center of gravity, and workability for mounting framing and other components.

6.2 Hull

6.2.1 Alternatives

For the hull, we will use either a single or double hull design. A single hull would sit a little higher on the water than a double hull, reducing overall drag, but it would have an increased surface area on the water and in the air. Another single hull design would use a surfboard-like shape to ride on top of the water. This design is being considered because of the availability of similar boards and the lower cost associated with building or purchasing one. The double hull design would consist of two hulls placed an equal distance away from the center of the hydrofoil, something like the hulls of a catamaran. This would possibly be more lightweight than a single hull design. Choosing the hull type will be directly related to minimum weight and drag due to the design of the rest of the craft.

6.2.2 Selection

Although it was initially determine that a catamaran style dual hull design would be best, budget and construction constraints forced a different decision. The design will now feature a single hull design derived from an old windsurfing board. This was able to be purchased much more reasonably and cuts down on the amount of weight that was expected to come from a two hull system. An advantage of this design rather than a monohull design, such as a kayak, is that it skims on the surface. Because drag is a function of the wetted surface area, decreasing the amount of hull in the water reduces the drag to overcome. Another advantage of the windsurfing board is that there are already holes and hardware that were once used for things like footholds. These holes can be used to mount framing for other components. Because these holes were once meant to hold the windsurfer on the board, they will be able to hold a similar load, albeit a different application.

6.3 Frame

6.3.1 Hydrofoil Support

6.3.1.1 Alternatives

Much of the design of the hydrofoil support, namely the strut, has been determined by the design of the drive train. The current design requires timing belts to run through the length of the rear strut, transmitting power from the pedaling motion to the propeller. This means that the strut needs to have a hollow section. The alternatives discussed here will be what materials to use for this construction.

The strut needs materials that are strong, lightweight, inexpensive, and can be formed to be aerodynamic for decreased drag as water flows around the strut. For strength, hollow metal rods for the timing belt housing were considered. Due to the size of the strut, a metal housing would mean significantly increased

weight. Next to be considered was carbon fiber tubing. Due to greater cost and decreased availability, further investigation needed to be done. Hollow PVC tubing is readily available in the engineering building.

Regardless of the housing of the timing belts, the strut needs to be aerodynamic. Hollow tubing is most readily available in round or square cross-sections. A round cross section can be used or form a symmetrical foil around this tube. This would be done in a similar fashion to the hydrofoil construction.

6.3.1.2 Selection

At this point, round hollow PVC tubing is the best design for the timing belt housing in the hydrofoil strut. Work is being done to reduce the necessary diameter for the tubing, thus reducing drag. The tubing must be able to withstand both the vertical compression of the rider and craft on the hydrofoil and the moment induced by the propeller.

In addition to using PVC, a symmetrical foil around the strut has been chosen for several reasons. For one, there will already be access to the necessary materials, as hydrofoil construction will be going on concurrently. Also, forming the strut in this shape will significantly decrease drag, and it is a worthwhile investment of time and energy. This can be done for any size of tubing. A profile has been chosen and it only needs to be scaled appropriately for the given tubing diameter.

6.3.2 Seat

6.3.2.1 Alternatives

Two main seat types have been considered. The first would be similar to mounting an existing bicycle to the watercraft. This design would provide a seat and also the mounting for the crank set for the drive train. The second design is to use a seated position similar to those implemented in recumbent bicycles. A recumbent position would provide more comfort for the rider yet still allows for efficient transmission of power from the legs to the pedals.

6.3.2.2 Selection

The recumbent design has been chosen for this application because it provides a much lower center of mass. With stability being an issue it is important to choose a design that minimizes concern for tipping over. Because this sort of design has been implemented in other pedal power applications, the fit for this project is natural.

7 Propulsion

7.1 General Requirements

The propulsion system is intricately involved with each component of the watercraft. For this reason there are many requirements that are specific to each design option and configuration. However, in general the propulsion system must satisfy the following:

- Appropriate gear reduction from pedals to propeller
- Translate axis of rotation 90 degrees from pedals to propeller
- Propeller generates sufficient thrust for hydrofoil lift
- Lightweight

The propulsion system is broken down into two major components, propeller and drive train. The alternative design options for each of these components and how they satisfy these requirements are discussed in the following sections.

7.2 Propeller

7.2.1 Alternatives

The two primary alternatives for a propeller deal with where the propeller is located. The first solution is to use a water propeller, and the second is to use an air propeller. However, there are problems related to each of these.

The problem with water propellers is cavitation. Cavitation can occur when the pressure on one side of a propeller becomes so low that water vaporizes. This causes the propeller to “slip”, meaning that efficiency is lowered and propulsion force decreases. This is a major concern as efficient propulsion is essential to keeping the boat up on hydrofoils.

The problem with air propellers is size. In order to create sufficient propulsion force, an air propeller has to be much larger than one in water. In turn this increases the cost and weight of the boat. The increased size also brings up safety concerns. A large exposed air propeller could injure a rider, and this is not ideal as the intended purpose of the boat is recreation.

7.2.2 Selection

The propeller will be one that was donated by Nauticraft Corporation. This propeller is capable of pushing a 500 lb boat up to 6 miles per hour. As our craft will have less weight and water displacement; the propeller should be able to produce the required thrust for hydrofoil lift and faster speeds. However, in the unlikely case that the propeller is not sufficient, a back-up plan is in place. Similar crafts have proven that a small prop from a standard 2 HP outboard motor can be used. Therefore, the design of the watercraft will incorporate an easy way to switch propellers in the event that the first fails.

7.3 Drive Train

7.3.1 Alternatives

The purpose of the drive train is to transfer power from the craft's pedals to the propeller. Figure 8 below is a sketch of the general boat layout and path of power transmission based on design selections made in previous sections.

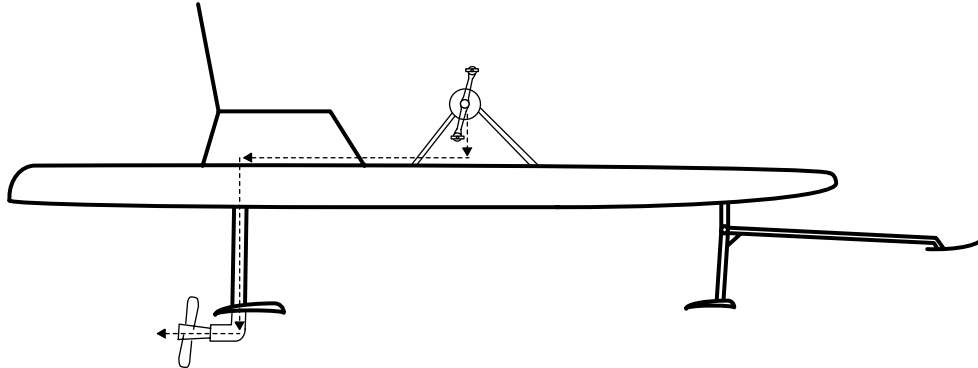


Figure 8: Watercraft Layout and Path of Power Transmission

The path of power transmission that the drive train must follow begins with a rider rotating the pedals. From here power moves down and towards the rear of the craft until it reaches the hydrofoil strut located under the seat. The path then turns downward and power is transmitted inside the hydrofoil strut to the propeller. A course through the hydrofoil strut is being used as to not introduce any more mechanisms into the water that cause additional drag.

There are two main tasks that the drive train must accomplish in the transmission from the pedals to the propeller. Previously mentioned as general requirements these tasks are appropriate gear reduction and translating axis of rotation 90 degrees between the pedals and prop. There are a variety of ways to accomplish these tasks, and the following presents the most viable alternatives.

The first task of gear reduction is needed so that the propeller spins at the optimum rate for creating thrust. Based on similar watercraft and specifics of chosen prop, the ratio of pedal rotation to that of the propeller varies anywhere from 1/4 up to 1/8. The design options for achieving gear reduction are the following:

- Bicycle chain and sprockets
- Belts and Pulleys
 - V
 - Flat
 - Timing
- Spur Gears

The second task of translating axis of rotation is needed because the propeller must spin perpendicular to the pedals. The practical options for achieving this are shown in the figure below.

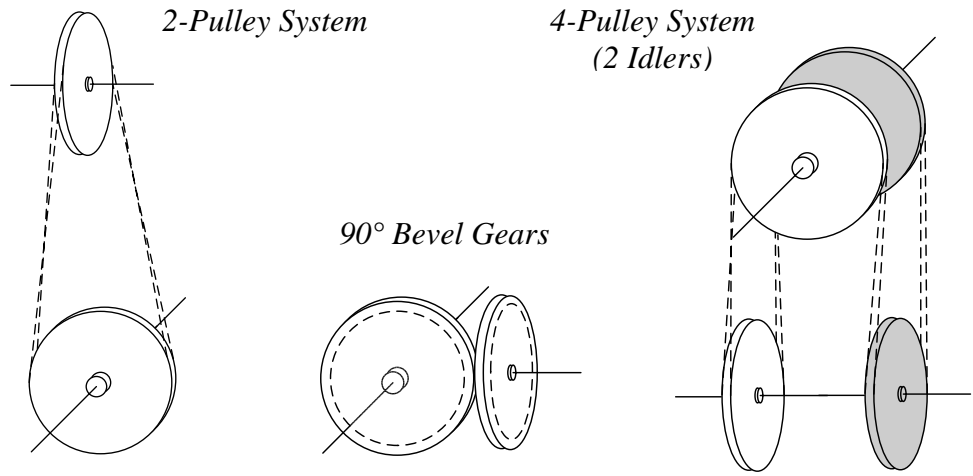


Figure 9: Methods for Transmitting Power Through a 90 Degree Turn

Note that the options are depicted with equal size pulleys or gears; if the sizes are varied, both axis translation and gear reduction could be achieved at the same time. Also, recalling the listed options for just gear reduction, the sizes can be varied so that no reduction occurs and they are only used for moving power over a certain distance. Given all of these variations there are a number of configurations for completing the power transmission path from the pedals to the propeller. The next figure shows a number of considered configurations.

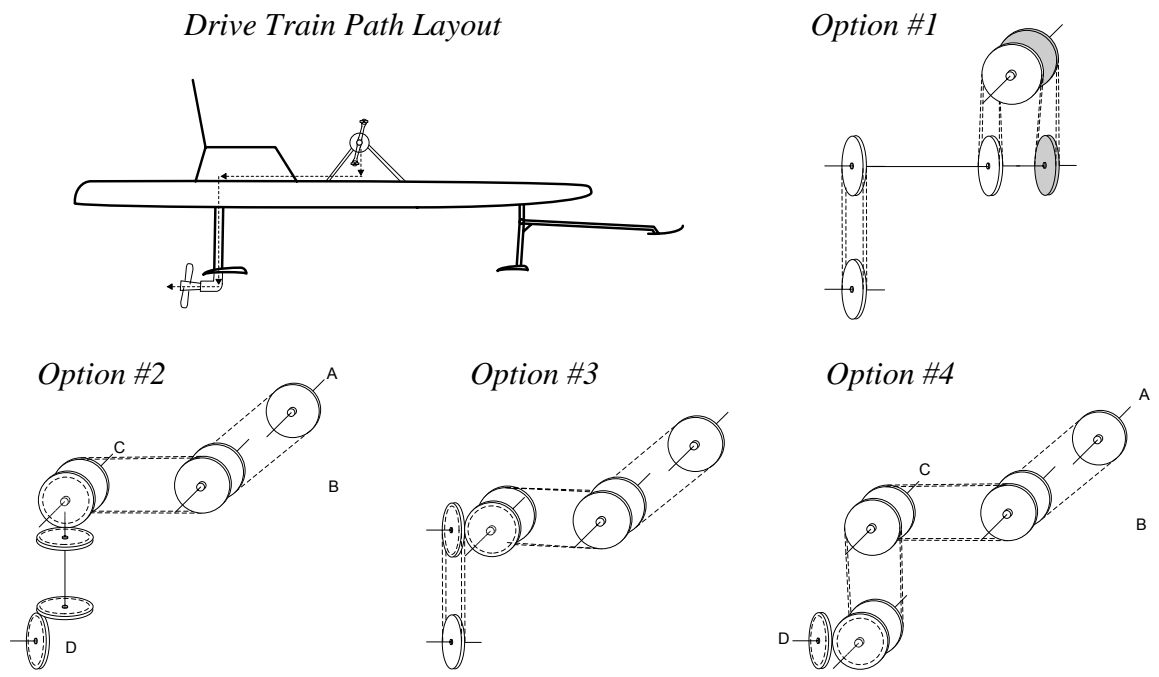


Figure 10: Possible Drive Train Layouts

7.3.2 Selection

As there are so many drive train configurations, selection is difficult. The rationale leading to the final selection is the following:

First, the 2-Pulley system for transferring power 90 degrees was eliminated. This is because as the belt is twisted between the pulleys there is added resistance. For this reason you do not see this method used in any of the options in Figure 10.

Next, both Option #2 and Option #5 are eliminated. These options have more components located at point D, shown in figure 10. This is the area at the bottom of the hydrofoil strut where the prop connects. It is here where designs need to remain as small and simple as possible, as to not increase size and drag.

Last, Option #1 is eliminated. This option is a very good solution to the 90 degree translation problem. However, the design and fabrication of the pedal drive unit is more complex compared to the other options. Also, final testing will require experimentation with what reduction ratios are optimal. Designing in a way to switch out pulleys makes this option even more complex.

Therefore, the drive train configuration selected is Option #3. In this design, the gear reduction will occur between points A and C, which is from the pedals to under the seat. This is so that varying the gear ratios in testing can be done in a place that is easily accessible. The gear reduction will be done using bicycle sprockets and chains. This is the choice instead of pulleys and belts or spur gears as sprockets and chains are designed for pedaling applications. They are more efficient, lightweight, adaptable, and inexpensive due to the fact we can salvage parts off of old bicycles.

After gear reduction, the 90 degree bevel gear option is used to transfer axis of rotation. As previously mentioned this option is used because of its simplicity. The bevel gears used will be a gearbox for the lower end of a small outboard motor. This is ideal because the housing already exists and will not have to be fabricated.

Last, power is transferred through the hydrofoil strut down to the propeller. For this bicycle sprockets and chains will not be used. This is because the smallest sprockets and chains available to us require the strut to be larger than it need to be. Therefore timing belts and pulleys are to be used. The timing option is used instead of V or flat as the toothed belt allows for greater torque in a smaller area. Also this is an area that could potential get wet, and V or flat belts are more likely to slip in these conditions. Finally, these belts and pulleys are very inexpensive and can be obtained through McMaster-Carr.

8 Conclusion

Through many hours of research, developing alternative designs, and consideration of non technical barriers, it has been determined that the project outlined in this proposal can be completed by May 2006. The goal remains to produce a human powered hydrofoil boat within the budget set forth by the Calvin College Engineering Department. This boat will have a hull made from an old windsurfer with a recumbent seat position with a set of pedals comfortably in front of the seated rider. One large hydrofoil will be located beneath the seat of the rider with two smaller hydrofoils located near the front of the hull to provide control and some lift. The hydrofoils will be the H105 profile and constructed of Styrofoam with a hard shell exterior made from fiberglass cloth and epoxy resin.

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Appendix A: Proposed Budget

| | |
|---------------------|-----------------|
| Body | \$130.00 |
| <i>Hull</i> | |
| Used Windsurfer | \$75.00 |
| <i>Frame</i> | |
| Aluminum Framing | \$0.00 |
| Connection Hardware | \$30.00 |
| Seat | \$5.00 |
| Paint | \$20.00 |
| Hydrofoil | \$220.00 |
| Styrofoam | \$40.00 |
| Fiberglass Cloth | \$50.00 |
| Epoxy | \$120.00 |
| Feeler Arm | \$10.00 |
| Propulsion | \$80.00 |
| Propeller | \$0.00 |
| <i>Drive Train</i> | |
| Shaft | \$10.00 |
| Crank Set | \$0.00 |
| Gearing | \$50.00 |
| Bearings | \$20.00 |
| Sprockets | \$0.00 |
| Steering | \$50.00 |
| Bearings | \$20.00 |
| Linkage/Controls | \$30.00 |
| Total | \$480.00 |

Appendix B: Detailed Hydrofoil Calculations – EES

```

"Hydrofoil Math"
g = 9.81 [m/s^2]
C_L_front = .4

C_L_rear = .4
C_d_front = 1
C_d_rear = .00784
rho_water = 998.23 [kg/m^3]
mu = .001002 [kg/m-s]
alpha = 1 [rad]
Chord_front = .25 [m]
"Chord_rear = .25 [m]"
"Length_front = 1 [m]"
"Length_rear = 2 [m]"
Thickness_front = .125*Chord_front
Thickness_rear = .125*Chord_rear
Base_AlumBeam = .00635 [m]
sigma_max_alum = 270*10^6
SafteyFactor = 2
sigma_max_alum = SafteyFactor*sigma_rear
A_L_front = Chord_front*Length_front
A_L_rear = Chord_rear*Length_rear
A_d_front = Length_front*Thickness_front

A_d_rear = Length_rear*Thickness_rear
v = 2.2352*10*(1/7)[m/s]
m_person = 250*0.45359237 [kg]
m_hull = 32.5*0.45359237 [kg]
m_drive = 20*0.45359237 [kg]
Length_Hull = 3.6576 [m]
x_Center = 1.778 [m]

x_front = 3.3528 [m]
percent_rear = .75

"Calculations"
F_w = g*(m_person + m_hull + m_drive)
I_rear = 1/12*Base_AlumBeam*Thickness_rear^3

Moment_rear = (Length_rear/2)^2*(F_L_rear/Length_rear)/2
sigma_rear = (Moment_rear*Thickness_rear/2)/I_rear
Re_rear = rho_water*v*Chord_rear/mu

"Lift"
F_L_front = 0.5*C_L_front*rho_water*v^2*A_L_front
F_L_rear = 0.5*C_L_rear*rho_water*v^2*A_L_rear

```

"acceleration due to gravity"

"lift coefficient - linked to angle of attack"

"drag coefficient"

"density of water @ 20 C"

"dynamic viscosity of water @ 20 C"

"angle of attack [radians]"

"area of front wing"

"area of rear wing"

"cross sectional area, perpendicular to flow"

"7 mph"

"12 ft"

"70 in. assume all of human mass is directly down at x_center"

"132 in. 1 ft back from front edge"

"assume all force is taken up by .25 in wide alum. beam height = thickness"

"Propulsion"

$$F_p = 1$$

"Drag"

$$F_{d_front} = 0.5 * C_{d_front} * \rho_{water} * v^2 * A_{d_front}$$

$$F_{d_rear} = 0.5 * C_{d_rear} * \rho_{water} * v^2 * A_{L_rear}$$

"hydrofoil locations"

$$\text{percent_rear} = F_{L_rear} / (F_{L_rear} + F_{L_front})$$

$$F_{L_rear} + F_{L_front} = F_w$$

$$F_w * x_{Center} - F_{L_front} * x_{front} - F_{L_rear} * x_{rear} = 0$$

"sum forces"

"sum Moments to get x_{rear} "

Solutions:

$$\alpha = 1 \text{ [rad]}$$

$$A_{L_front} = 0.1653 \text{ [m}^2\text{]}$$

$$\text{Chord}_{front} = 0.25 \text{ [m]}$$

$$C_{d_rear} = 0.00784$$

$$F_{d_front} = 105.2 \text{ [N]}$$

$$F_{L_rear} = 1010 \text{ [N]}$$

$$g = 9.81 \text{ [m/s}^2\text{]}$$

$$\text{Length}_{Hull} = 3.658 \text{ [m]}$$

$$\mu = 0.001002 \text{ [kg/m-s]}$$

$$m_{person} = 113.4 \text{ [kg]}$$

$$\rho_{water} = 998.2 \text{ [kg/m}^3\text{]}$$

$$\sigma_{rear} = 1.350E+08 \text{ [N/m}^2\text{]}$$

$$v = 3.193 \text{ [m/s]}$$

$$x_{rear} = 1.253 \text{ [m]}$$

$$A_{d_front} = 0.02066 \text{ [m}^2\text{]}$$

$$A_{L_rear} = 0.4959 \text{ [m}^2\text{]}$$

$$\text{Chord}_{rear} = 0.3038 \text{ [m]}$$

$$C_{L_front} = 0.4$$

$$F_{d_rear} = 19.79 \text{ [N]}$$

$$F_p = 1 \text{ [N]}$$

$$I_{rear} = 2.897E-08 \text{ [m}^4\text{]}$$

$$\text{Length}_{rear} = 1.633 \text{ [m]}$$

$$m_{drive} = 9.072 \text{ [kg]}$$

$$\text{percent}_{rear} = 0.75$$

$$\text{SafetyFactor} = 2$$

$$\text{Thickness}_{front} = 0.03125 \text{ [m]}$$

$$x_{Center} = 1.778 \text{ [m]}$$

$$A_{d_rear} = 0.06199 \text{ [m}^2\text{]}$$

$$\text{Base}_{AlumBeam} = 0.00635 \text{ [m]}$$

$$C_{d_front} = 1$$

$$C_{L_rear} = 0.4$$

$$F_{L_front} = 336.5 \text{ [N]}$$

$$F_w = 1346 \text{ [N]}$$

$$\text{Length}_{front} = 0.6612 \text{ [m]}$$

$$\text{Moment}_{rear} = 206 \text{ [N-m]}$$

$$m_{hull} = 14.74 \text{ [kg]}$$

$$Re_{rear} = 966365$$

$$\sigma_{max,alum} = 2.700E+08 \text{ [N/m}^2\text{]}$$

$$\text{Thickness}_{rear} = 0.03797 \text{ [m]}$$

$$x_{front} = 3.353 \text{ [m]}$$

Appendix C: Detailed Hydrofoil Calculations - Excel

| Inputs | |
|-------------------------|---------------|
| Chord Length: | 10 inches |
| Center Location Offset: | 0.18 inches |
| Thickness: | 0.0625 inches |
| Length: | 74.4 inches |
| Tensile Strength: | 12000 PSI |

| Outputs | |
|------------------------|---------------------------------|
| Area: | 1.248803125 inches ² |
| Total I _x : | 0.250234996 inches ⁴ |
| Shear Stress: | 340.2034989 psi |
| Max Loading: | 424.8471925 lb |

| Solid Outer Area | | | | | | | | | | Inside Solid Area, to be subtracted | | | | | | | | | | |
|------------------|--------------|---------------------|---------|----------------|---------------------|-------------|----------|--------------|----------------|-------------------------------------|--------------|---------------------|---------|----------------|---------------------|-----------|----------|--------------|----------------|----------|
| X coordinate | Y coordinate | Scaled X coordinate | Delta X | Median Delta X | Scaled Y coordinate | Area | R | R'area | I _x | X coordinate | Y coordinate | Scaled X coordinate | Delta X | Median Delta X | Scaled Y coordinate | Area | R | R'area | I _x | |
| 0.99645 | 0.00088 | 9.9645 | 0.0355 | 0.06755 | 0.0088 | 0.00058444 | -0.1756 | -0.000104384 | 1.8334E-05 | 0.99645 | 0.00088 | 9.9645 | 0.0355 | 0.06755 | -0.0337 | -0.00363 | -0.20685 | 0.000750335 | | |
| 0.98649 | 0.00353 | 9.8649 | 0.0096 | 0.12685 | 0.0096 | 0.004477605 | -0.16235 | -0.000726972 | 0.00011849 | 0.98649 | 0.00353 | 9.8649 | 0.0096 | 0.12685 | -0.0272 | -0.00345 | -0.1936 | 0.000667982 | | |
| 0.97108 | 0.00762 | 9.7108 | 0.1541 | 0.1797 | 0.0762 | 0.013689314 | -0.1419 | -0.001943057 | 0.00028235 | 0.97108 | 0.00762 | 9.7108 | 0.1541 | 0.1797 | 0.0137 | 0.002462 | -0.17315 | -0.00042628 | | |
| 0.95055 | 0.01271 | 9.5055 | 0.2053 | 0.2295 | 0.1271 | 0.029137675 | -0.11645 | -0.003393082 | 0.00043435 | 0.95055 | 0.01271 | 9.5055 | 0.2053 | 0.2295 | 0.0646 | 0.014181 | -0.1477 | -0.00218737 | | |
| 0.92523 | 0.01867 | 9.2523 | 0.2532 | 0.27485 | 0.1867 | 0.051314446 | -0.08665 | -0.004464601 | 0.00053434 | 0.92523 | 0.01867 | 9.2523 | 0.2532 | 0.27485 | -0.1242 | 0.0043136 | -0.1179 | -0.00402468 | | |
| 0.89558 | 0.0253 | 8.9558 | 0.2956 | 0.3158 | 0.253 | 0.07989974 | -0.0535 | -0.004274511 | 0.00065487 | 0.89558 | 0.0253 | 8.9558 | 0.2956 | 0.3158 | 0.1905 | 0.06016 | -0.08475 | -0.00509855 | 0.000614 | |
| 0.86207 | 0.03236 | 8.6207 | 0.3351 | 0.36225 | 0.3236 | 0.1138881 | -0.0192 | -0.002074583 | 0.00103246 | 0.86207 | 0.03236 | 8.6207 | 0.3351 | 0.36225 | 0.3332 | 0.091972 | -0.04945 | -0.00454804 | 0.000747 | |
| 0.82513 | 0.03957 | 8.2513 | 0.3694 | 0.38435 | 0.3657 | 0.152087296 | 0.01785 | 0.002714758 | 0.00203293 | 0.82513 | 0.03957 | 8.2513 | 0.3694 | 0.38435 | 0.3694 | 0.128065 | -0.0134 | -0.00171608 | 0.001206 | |
| 0.7852 | 0.04667 | 7.852 | 0.3993 | 0.4122 | 0.4667 | 0.19237374 | 0.05335 | 0.010263139 | 0.00403926 | 0.7852 | 0.04667 | 7.852 | 0.3993 | 0.4122 | 0.4042 | 0.166611 | 0.0221 | 0.00368210 | 0.00235 | |
| 0.74269 | 0.0534 | 7.4269 | 0.4251 | 0.4359 | 0.534 | 0.2327706 | 0.087 | 0.020251042 | 0.00729317 | 0.74269 | 0.0534 | 7.4269 | 0.4251 | 0.4359 | 0.4715 | 0.205527 | 0.0575 | 0.01458122 | 0.00446 | |
| 0.69802 | 0.05955 | 6.9802 | 0.4467 | 0.4554 | 0.5955 | 0.2711907 | 0.11775 | 0.031932705 | 0.01177422 | 0.69802 | 0.05955 | 6.9802 | 0.4467 | 0.4554 | 0.533 | 0.242728 | 0.0865 | 0.02095989 | 0.007563 | |
| 0.65161 | 0.06492 | 6.5161 | 0.4641 | 0.47085 | 0.6492 | 0.30567582 | 0.1446 | 0.044200724 | 0.01712729 | 0.65161 | 0.06492 | 6.5161 | 0.4641 | 0.47085 | 0.5867 | 0.276248 | 0.11335 | 0.031312676 | 0.011473 | |
| 0.60385 | 0.06934 | 6.0385 | 0.4776 | 0.48225 | 0.6934 | 0.33439215 | 0.1667 | 0.055743171 | 0.02269046 | 0.60385 | 0.06934 | 6.0385 | 0.4776 | 0.48225 | 0.6309 | 0.304252 | 0.13545 | 0.041210869 | 0.015674 | |
| 0.55516 | 0.0727 | 5.5516 | 0.4869 | 0.4895 | 0.727 | 0.3558965 | 0.1835 | 0.065301503 | 0.02756864 | 0.55516 | 0.0727 | 5.5516 | 0.4869 | 0.4895 | 0.6645 | 0.325273 | 0.15225 | 0.049522776 | 0.019509 | |
| 0.50595 | 0.07493 | 5.0595 | 0.4921 | 0.4927 | 0.7493 | 0.36918011 | 0.19465 | 0.071860908 | 0.03126076 | 0.50595 | 0.07493 | 5.0595 | 0.4921 | 0.4927 | 0.6668 | 0.338386 | 0.1634 | 0.05292331 | 0.022336 | |
| 0.45662 | 0.07748 | 4.5662 | 0.4933 | 0.4916 | 0.76 | 0.373616 | 0.2 | 0.0747232 | 0.03292802 | 0.45662 | 0.07748 | 4.5662 | 0.4933 | 0.4916 | 0.6975 | 0.342891 | 0.16875 | 0.057862856 | 0.023666 | |
| 0.40763 | 0.07954 | 4.0763 | 0.4899 | 0.4858 | 0.7595 | 0.3689561 | 0.19975 | 0.073700779 | 0.03245789 | 0.40763 | 0.07954 | 4.0763 | 0.4899 | 0.4858 | 0.697 | 0.338603 | 0.1685 | 0.057054538 | 0.023322 | |
| 0.35946 | 0.07844 | 3.5946 | 0.4817 | 0.47525 | 0.7484 | 0.3556771 | 0.1942 | 0.069072493 | 0.03001518 | 0.35946 | 0.07844 | 3.5946 | 0.4817 | 0.47525 | 0.6859 | 0.325974 | 0.16295 | 0.053171459 | 0.021435 | |
| 0.31258 | 0.07722 | 3.1258 | 0.4688 | 0.46975 | 0.7272 | 0.3343302 | 0.1836 | 0.061380205 | 0.02800329 | 0.31258 | 0.07722 | 3.1258 | 0.4688 | 0.46975 | 0.6647 | 0.305956 | 0.15235 | 0.046557524 | 0.018465 | |
| 0.26751 | 0.06966 | 2.6751 | 0.4507 | 0.4394 | 0.6966 | 0.30608604 | 0.1683 | 0.051514281 | 0.02104725 | 0.26751 | 0.06966 | 2.6751 | 0.4507 | 0.4394 | 0.6341 | 0.278624 | 0.13705 | 0.038185356 | 0.014569 | |
| 0.2247 | 0.0657 | 2.247 | 0.4281 | 0.4145 | 0.657 | 0.2723265 | 0.1485 | 0.040440485 | 0.0158012 | 0.2247 | 0.0657 | 2.247 | 0.4281 | 0.4145 | 0.5945 | 0.24642 | 0.11725 | 0.028892774 | 0.010645 | |
| 0.18461 | 0.06093 | 1.8461 | 0.4009 | 0.38525 | 0.6093 | 0.234732825 | 0.12465 | 0.029259447 | 0.0109017 | 0.18461 | 0.06093 | 1.8461 | 0.4009 | 0.38525 | 0.5468 | 0.210655 | 0.0934 | 0.019675149 | 0.007086 | |
| 0.14275 | 0.0554 | 1.4275 | 0.3696 | 0.3505 | 0.554 | 0.1950357 | 0.097 | 0.018918463 | 0.00682339 | 0.14275 | 0.0554 | 1.4275 | 0.3696 | 0.3505 | 0.4915 | 0.173033 | 0.06575 | 0.01373692 | 0.004231 | |
| 0.1142 | 0.0492 | 1.142 | 0.3345 | 0.3153 | 0.492 | 0.1551276 | 0.066 | 0.010238422 | 0.00380497 | 0.1142 | 0.0492 | 1.142 | 0.3345 | 0.3153 | 0.4295 | 0.135421 | 0.03475 | 0.004705892 | 0.002245 | |
| 0.08459 | 0.0442 | 0.8459 | 0.2981 | 0.2756 | 0.4242 | 0.11699952 | 0.0321 | 0.003752796 | 0.00187358 | 0.08459 | 0.0442 | 0.8459 | 0.2981 | 0.2756 | 0.3617 | 0.099685 | 0.00085 | 0.003718E-05 | 0.001087 | |
| 0.05928 | 0.03517 | 0.5928 | 0.2551 | 0.2334 | 0.3517 | 0.06208678 | -0.00415 | -0.00034066 | 0.00094754 | 0.05928 | 0.03517 | 0.5928 | 0.2551 | 0.2334 | 0.2892 | 0.067499 | -0.0354 | -0.00238947 | 0.000555 | |
| 0.03791 | 0.0276 | 0.3791 | 0.18915 | 0.176 | 0.276 | 0.0252054 | -0.042 | -0.002192671 | 0.00042349 | 0.03791 | 0.0276 | 0.3791 | 0.18915 | 0.176 | 0.1815 | 0.2135 | 0.040384 | -0.07325 | -0.0025805 | 0.000374 |
| 0.02125 | 0.01989 | 0.2125 | 0.1666 | 0.1432 | 0.1989 | 0.02848248 | -0.08055 | -0.002294264 | 0.0002787 | 0.02125 | 0.01989 | 0.2125 | 0.1666 | 0.1432 | 0.1364 | 0.169532 | -0.1118 | -0.00218373 | 0.000274 | |
| 0.00927 | 0.01229 | 0.0927 | 0.1198 | 0.09565 | 0.1229 | 0.011753589 | -0.11855 | -0.001393601 | 0.00018001 | 0.00927 | 0.01229 | 0.0927 | 0.1198 | 0.09565 | 0.0604 | 0.005777 | -0.1498 | -0.00086543 | | |
| 0.00011 | 0.00013 | 0.0011 | 0.0011 | 0.00013 | 0.0011 | 0.000246225 | -0.16775 | -4.13042E-05 | 6.9411E-06 | 0.00011 | 0.00013 | 0.0011 | 0.0011 | 0.00013 | -0.038 | -0.00038 | -0.199 | 7.59981E-05 | | |
| 0.00003 | -0.00044 | 0.0003 | 0.0003 | 0.0008 | 0.0003 | 0.000026997 | -0.17535 | -4.72919E-06 | 8.2946E-07 | 0.00003 | -0.00044 | 0.0003 | 0.0003 | 0.0008 | 0.00008 | -0.00165 | -0.0591 | -8.6E-05 | 0.15095 | |
| 0.00044 | -0.00175 | 0.0044 | -0.0041 | -0.0063 | -0.0175 | 0.00011025 | -0.18875 | -2.08097E-05 | 3.9306E-06 | 0.00044 | -0.00175 | 0.0044 | -0.0041 | -0.0063 | 0.045 | -0.00028 | -0.1575 | 4.46E-13E-05 | | |
| 0.00129 | -0.00313 | 0.0129 | -0.0085 | -0.0313 | -0.0129 | 0.000674515 | -0.19565 | -0.000319698 | 2.5875E-05 | 0.00129 | -0.00313 | 0.0129 | -0.0085 | -0.0313 | 0.0129 | -0.00085 | -0.02102 | -0.1644 | 0.000110536 | |
| 0.00475 | -0.00659 | 0.0475 | -0.0346 | -0.0681 | -0.0659 | 0.00448779 | -0.21295 | -0.000956575 | 0.00020514 | 0.00475 | -0.00659 | 0.0475 | -0.0346 | -0.0681 | 0.0034 | 0.000232 | -0.1817 | -4.2071E-05 | | |
| 0.01491 | -0.01268 | 0.1491 | -0.1016 | -0.1272 | -0.1268 | 0.01612896 | -0.2434 | -0.003825789 | 0.00097715 | 0.01491 | -0.01268 | 0.1491 | -0.1016 | -0.1272 | 0.0043 | 0.000819 | -0.21215 | -0.00173517 | | |
| 0.03019 | -0.01888 | 0.3019 | -0.1528 | -0.1769 | -0.1888 | 0.03339872 | -0.2744 | -0.009164609 | 0.00261398 | 0.03019 | -0.01888 | 0.3019 | -0.1528 | -0.1769 | -0.1263 | 0.022342 | -0.24315 | -0.00543257 | 0.001351 | |
| 0.05029 | -0.02499 | 0.5029 | -0.201 | -0.22355 | -0.2499 | 0.05585145 | -0.30495 | -0.017036078 | 0.00548588 | 0.05029 | -0.02499 | 0.5029 | -0.201 | -0.22355 | -0.1874 | 0.041893 | -0.2737 | -0.01146819 | 0.003261 | |
| 0.0749 | -0.03081 | 0.749 | -0.2672 | -0.3081 | -0.3081 | 0.08232432 | -0.33405 | -0.027500439 | 0.00983775 | 0.0749 | -0.03081 | 0.749 | -0.2672 | -0.3081 | -0.2456 | 0.065624 | -0.3028 | -0.00987104 | 0.006347 | |
| 0.10373 | -0.03617 | 1.0373 | -0.2883 | -0.3079 | -0.3617 | 0.11136743 | -0.36085 | -0.040186397 | 0.01571561 | 0.10373 | -0.03617 | 1.0373 | -0.2883 | -0.3079 | -0.2992 | 0.092124 | -0.3296 | -0.03036396 | 0.010695 | |
| 0.13648 | -0.04089 | 1.3648 | -0.3275 | -0.3456 | -0.4089 | 0.14131584 | -0.38445 | -0.054328875 | 0.02285573 | 0.13648 | -0.04089 | 1.3648 | -0.3275 | -0.3456 | -0.3464 | 0.119716 | -0.3532 | -0.04228363 | 0.016132 | |
| 0.17285 | -0.04482 | 1.7285 | -0.3637 | -0.38005 | -0.4482 | 0.17033841 | -0.4041 | -0.068833751 | 0.03066723 | 0.17285 | -0.04482 | 1.7285 | -0.3637 | -0.38005 | -0.3857 | 0.146585 | -0.37285 | -0.0545432 | 0.022195 | |
| 0.21249 | -0.04786 | 2.1249 | -0.3964 | -0.411 | -0.4786 | 0.1967046 | -0.4193 | -0.082478239 | 0.03833786 | 0.21249 | -0.04786 | 2.1249 | -0.3964 | -0.411 | -0.4161 | 0.171017 | -0.38805 | -0.06363919 | 0.02822 | |
| 0.25505 | -0.04992 | 2.5505 | -0.4256 | -0.4383 | -0.4992 | 0.21879936 | -0.4296 | -0.093996205 | 0.04492451 | 0.25505 | -0.04992 | 2.5505 | -0.4256 | -0.4383 | -0.4367 | 0.191406 | -0.39835 | -0.0724642 | 0.033415 | |
| 0.30015 | -0.05094 | 3. | | | | | | | | | | | | | | | | | | |