Team 6: Calvin Supermileage

Project Proposal Feasibility Study

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Engineering 339 Senior Design Project
Calvin College

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Executive Summary:

The Calvin College 2011-2012 senior design Team 6: Supermileage includes the following senior engineers: Ben Beezhold (ME), Jon Hofman (ME), David Kaemingk (ME and CivE), Will Vanden Bos (ME) and Jacob Vriesema (ME). In this report the team proposes design solutions for creating a single-occupant high fuel-mileage car to compete in the annual SAE Supermileage® competition in Marshall, MI. In such a design, the team resolves to minimize inefficiencies that arise from tire rolling resistance, aerodynamic drag, and engine performance. The following report outlines project feasibility as well as highlights design solutions that encompass aerodynamics, engine optimization, frame design, steering design and power transmission all while observing SAE Supermileage® competition rules.
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1 Introduction

On July 29, 2011, President Obama announced that automakers have agreed to increase car fuel efficiency to 54.5 mpg by 2025 (NHTSA). A drastic increase over today’s efficiency standards, this move underscores an increasing need to reduce the demand for fossil fuels. Pessimistic analysis of worldwide supply estimates that peak oil production has already occurred (Alekleit). With alternative energy vehicles presently absent on showroom floors, engineers need little convincing of the significance of highly efficient cars. It is in this light that the Society of Automotive Engineers (SAE) sponsors the SAE Supermileage® competition. Engineering students from across North America design a high efficiency, single-occupant vehicle: the best combination of fuel economy, written and oral presentation wins. The team represented here proposes to design and build a new vehicle to participate in this competition for their Senior Design Project. A few broad outcomes of such a project include overcoming design difficulties, expansion of knowledge in respective design niches, and interpersonal group management.

The Calvin College senior design project is the capstone project for all senior engineering students at Calvin College. The class is divided into two sections: ENGR 339 during the Fall Semester and ENGR 340 during the Spring Semester. Class time in ENGR 339 is spent teaching senior engineering students how to work as a team. Lectures and discussions address topics such as communication, safety and project management. The following sections investigate the feasibility of this project and recommend the criteria necessary to complete this project.

2 Project Management

2.1 Team Organization

The Calvin College Supermileage senior design team consists of five mechanical engineers. Design and analysis work was divided into five subcategories based on team members’ skills, interests, and previous internship experience. Figure 1, below, shows the organization chart of Calvin Supermileage along with supporting faculty and mentors.
2.1.1 Team Member Roles in Subcategories

**Ben Beezhold:** looks forward to the upcoming challenges in designing a vehicle to compete with other large, engineering schools. While his engineering passion lies within aerospace, this design project provides a quintessential combination of aerodynamics, structures, and practical shop engineering. Returning to Grand Rapids after a summer internship at Boeing in Long Beach, CA, Ben hopes to apply the skills he attained with regards to group organization, leadership, and structural analysis. His efforts on this project will be directed towards developing and analyzing the aerodynamic model and the designing of the vehicle frame structure.

**Jon Hofman:** is interested to learn about the manifold engineering aspects of constructing a motor vehicle. As a two-year intern with Innotec in Zeeland, MI, Jon has been intimately involved with the design, fabrication, and optimization of machines. Combining skills from the machine design classroom and the shop floor, Jon will be a part of the Engine Improvement sub-group and the Optimization/Gearing/Transmission sub-group.

**David Kaemingk:** has long been interested in structural bodies in dynamic machines and energy conservation. He has a passion for using technology to aid in sustainable practices. He has background in
structural and mechanical engineering, experience in FEA and 3D-CAD modeling, and the desire to research engine optimization. For these reasons David will be primarily working with the Frame/Materials and Engine Improvements subgroups. David will also take a lead role in group communication and coordination.

**Will Vanden Bos:** looks forward to working on a team developing a more fuel-efficient vehicle. He is passionate about sustainable energy practices, especially with regard to the automotive industry. Will has extensive CATIA experience in addition to industry experience working with and testing machinery. Because of this, he will be working mainly with the Frame/Materials and Optimizing/Gearing/Transmission groups. Will also plans to take a lead role in research and communicating findings to the rest of the group.

**Jacob Vriesema:** has long been interested in fuel efficiency. Writing papers throughout high school and college on this and similar topics has gained him a well-rounded overview of the current technologies that exist as well as the current design barriers preventing further research. Jacob has been a part of many hands-on building projects and knows how to use many tools efficiently and to the best of their capabilities. He also is very organized and wishes to keep all members up to date with all changes pertaining to the documentation and design of the vehicle. Jacob will work primarily with the Aerodynamics/Body and the Frame/Materials sub-groups as well as take a lead role in project documentation.

2.1.2 **Documentation Techniques**

Accountability for reaching project goals often took the form of informal meetings three days a week. Team meetings were held each weekend. In the early stages of the project, these meetings mostly consisted of brainstorming sessions and research. Additional meetings were often called in order to prepare various assignments for the Engineering 339 class. For each meeting, minutes were recorded and saved in the team folder on one of Calvin’s Servers (Engr Scratch Drive). Note that all project documents were saved in this folder.

2.2 **Scheduling**

The Engineering 339 course schedule and assignments served as the framework for scheduling tasks throughout the duration of this project. Course assignments helped move the project from concept to completion. With such assignments serving as the framework, each task was assigned an estimated length of time for completion. These assignments provided a schedule with concrete deadlines and also served as project milestones. Free software called “OpenProj” assisted in schedule management. Using OpenProj, the team kept track of tasks with a Gantt Chart. The goal here was to break the milestones into tasks with a maximum execution time of 8 hours. This helped emphasize the critical essence of time. Task descriptions and time estimates were edited periodically to remain consistent with actual team progress. The most recent schedule update required several tasks to be moved ahead in time since others had not yet been completed. Latent tasks included wind tunnel testing and frame design analysis. It is anticipated that these delays will not affect the final outcome of the project.
2.3 **Budget Management**

The crux of every project is its budget. Will Vanden Bos has taken charge of managing this important information and has developed projections for project costs. Section 5.3 (Cost Estimation) provides an in-depth look at estimated costs and budgeting. A spreadsheet stored on the Scratch Drive (Engr Scratch Drive) contains all budget data. The budget will become increasingly important as the team compares tradeoffs for project material and equipment. Careful spending and control of funds underscores one of the integral values of this team: stewardship.

2.4 **Design Norms**

As Christian Engineers, we have a calling to create designs that have significance and fulfill the cultural mandate. In approaching this topic, we believe that the design of a high-mileage vehicle should exhibit the following principals.

*Stewardship:* The overall intention of the vehicle should demonstrate careful use of the Earth’s resources. The entire project is framed by this idea of stewardship as we strive to alleviate dependence on fossil fuels.

*Integrity:* As a team, we strive to promote honesty and professionalism interacting with each other, the professors, and any outside companies and consultants. We hope to display good character and sportsmanship throughout the duration of the project, especially at the Supermileage® competition.

*Transparency:* By maintaining an open stream of communication with all who are involved in this project, we strive to produce consistent, reliable, and predictable solutions.

2.5 **Method of Approach**

Maximizing the fuel efficiency of the vehicle can be addressed in the following ways:

1. Minimizing the resistances acting on the vehicle
2. Maximizing the engine’s efficiency
3. Optimizing vehicle operation in accordance to competition requirements

Because of time and budget constraints, team Supermileage performed a preliminary analysis to identify the factors that have the greatest potential to affect fuel economy. This analysis sets a basis for project direction and more in-depth analysis and testing.

2.5.1 **Minimizing Resistances**

The significant inefficiencies opposing the vehicle motion are wind resistance, rolling resistance of wheels, bearing resistance, and power transmission losses. These are depicted as equivalent forces in Figure 2.
Figure 2: Forces acting on vehicle during operation

A major aspect of the 2011-12 Supermileage team is formulation of a design that minimizes these resistances. An order of magnitude analysis was performed for the resisting forces using Equations 1-3.

\[ F_{drag} = C_d A_{proj} \frac{V^2}{2} \]  
\[ F_{rr} = C_{rr} V \]  
\[ F_{bearing} = C_b F_N \]

Eqn. 1  
Eqn. 2  
Eqn. 3

Note that \( C_d, C_{rr}, \) and \( C_b \) are coefficients of drag, rolling resistance, and bearing resistance respectively. Variables \( A_{proj}, F_N, \) and \( V \) represent the projected frontal area of the vehicle, normal force, and vehicle velocity. All these combine to form design variables for this project. By analyzing the tradeoffs between these variables, studies may be done to determine how to minimize resistances. The values which minimize resistances will then be used in the design of the vehicle.

The team examined tradeoffs between the above forces and vehicle velocity. Results showed that at lower velocities, resistance due to rolling resistance dominated. However, at a certain velocity (approximately 23mph) losses from aerodynamic drag dominate. Figure 3 shows each resistance as a function of velocity.
Assumptions made in this analysis are displayed in Table 1. Power loses were also calculated to illuminate efficiency “offenders.” Note that the estimated range column specifies a maximum and minimum value for each variable. The estimated power loss range column lists the power losses associated with the maximum and minimum values. The difference in power loss column represents the difference between the maximum and minimum values and therefore represents the sensitivity of that variable to power loss.
Table 1: Key design variables

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Mass [kg]</td>
<td>$M_{\text{vehicle}}$</td>
<td>52.2</td>
<td>40 - 90</td>
<td>63</td>
<td>14</td>
</tr>
<tr>
<td>Vehicle Velocity [m/s]</td>
<td>$V$</td>
<td>6.7</td>
<td>6.7 - 11.2</td>
<td>69</td>
<td>94</td>
</tr>
<tr>
<td>Coefficient of Drag</td>
<td>$C_d$</td>
<td>0.12</td>
<td>0.1 - 0.25</td>
<td>66</td>
<td>20</td>
</tr>
<tr>
<td>Projected Area [m$^2$]</td>
<td>$A_{\text{proj}}$</td>
<td>0.75</td>
<td>0.5 - 1</td>
<td>63</td>
<td>11</td>
</tr>
<tr>
<td>Rolling Resistance Coefficient</td>
<td>$C_{rr}$</td>
<td>0.0055</td>
<td>0.005 -0.01</td>
<td>65</td>
<td>37</td>
</tr>
<tr>
<td>Coefficient of Bearing Friction</td>
<td>$C_b$</td>
<td>0.0005</td>
<td>0.0001 - 0.001</td>
<td>60</td>
<td>3</td>
</tr>
</tbody>
</table>

This analysis suggests that aerodynamic drag and rolling resistance will have the greatest effect on the car while bearing resistance is less significant and therefore Team Supermileage will focus on reducing aerodynamic drag and rolling resistance.

The easiest and most effective way to reduce aerodynamic drag is to simply reduce the vehicle’s operating velocity. However, other factors which effect aerodynamic drag are more difficult to quantify using basic calculations. Because of this, the team plans on conducting basic wind tunnel testing in order to help guide aerodynamic decision making.

At low velocities, rolling resistance is the largest resistance on the vehicle. Therefore much time will be devoted to deciding wheel components (e.g. wheel diameter, width, and tire pressure) and minimizing vehicle weight.

The preliminary resistance calculations described above do not consider the effects of wheel alignment or transmission losses. However, these factors are still expected to be significant and will require design work—steering and power transmission considerations are discussed in greater detail in their respective sections.
2.5.2 Maximizing Engine Efficiency

Preliminary calculations suggest that increasing the efficiency of the engine by 1 percent will result in an increase of fuel economy by roughly 86 mpg (see Appendix B for calculations). For this reason, Team Supermileage will spend significant time maximizing the fuel efficiency of the engine. The team plans on constructing a dynamometer coupled with fuel and air flow rate instrumentation that will be used to find the vehicles optimum operating rpm. This will also help test modifications made to the engine.

2.5.3 Optimizing Vehicle Operation

The competition requirements state that the vehicle must maintain an average velocity between 15mph and 25mph. Bearing resistance and aerodynamic drag are proportional with velocity and thus are minimized when velocity is minimized. However, maintaining 15mph may require the engine to operate at a less than ideal rpm. Many successful teams in the past have approached this problem by fine tuning gearing in the transmission as well as coasting and accelerating alternately. This allows the engine to operate around its most efficient rpm. Identifying the optimum gearing and coast and burn pattern will be heavily dependent on dynamometer testing results.

3 Task Specification and Schedule

As previously mentioned, team members were assigned tasks based on previous skills, interests, and internship experience. Often, general tasks (e.g. paper writing or research) were completed on a volunteer basis. The following subsections take a closer look at how the project was divided and provides estimates of each task’s completion time.

3.1 Project Breakdown

In working towards the ultimate goal of high fuel efficiency, the team divided major vehicle components into subsystems. This helped clarify the design and research needs. Subsystems included the following: Engine Integration, Aerodynamics Design, Frame Design, and Power Transmission. Figure 4 shows visually the project breakdown.
3.1.1 Engine Integration

The Engine Integration Team will handle engine modifications and engine testing procedures. Most importantly, this group will work to build a dynamometer device that will record the amount of fuel used for different engine power outputs, driving efficiency and performance optimization. A system architecture diagram is shown below in Figure 5.

3.1.2 Aerodynamics
The Aerodynamics group will focus primarily on devising test methods to confirm design decisions relating to decreasing aerodynamic drag. Wind tunnel tests are the primary means by which this will be accomplished. In addition, this group will decide which material ought to be used for the vehicle shell (i.e. covering). Figure 6 shows the system architecture for the aerodynamics subsystem.

3.1.3 Frame Design

Team members working on the frame design subsystem are concerned primarily with creating a lightweight yet durable frame. Tradeoffs for this subsystem included balancing material properties and costs. Figure 7 shows a diagram of this subsystem’s architecture. Steering design and wheel selection also took place in this group.
3.1.4 Power Transmission

While the engine integration group is looking at modifications to the engine, the power transmission group will focus on how to efficiently transmit power from the engine to the wheels. Shifting speeds and coast/burn techniques are also being considered in this group. Figure 8 shows the subsystem architecture for the power transmission group.
3.2 Gantt Chart and Tasks

A complete list of project task specifications is given in Table 2, below. Figure 9 shows an updated work breakdown schedule (as of December 9, 2011).

Figure 9: Work breakdown schedule with Gantt chart
Table 2: List of project tasks with estimated time allocations

<table>
<thead>
<tr>
<th>TASK NAME</th>
<th>HOURS PER PERSON</th>
</tr>
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<tbody>
<tr>
<td><strong>Engine Work:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Engine Research:</strong></td>
<td></td>
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<tr>
<td>Research designs from previous teams</td>
<td>6 hours</td>
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<tr>
<td>Research fuel injection</td>
<td>6 hours</td>
</tr>
<tr>
<td>Research Increasing compression ratio</td>
<td>6 hours</td>
</tr>
<tr>
<td>Research carburetor devices</td>
<td>6 hours</td>
</tr>
<tr>
<td>Begin constructing a dynomometer</td>
<td>10 hours</td>
</tr>
<tr>
<td><strong>Engine Implementation:</strong></td>
<td></td>
</tr>
<tr>
<td>Test engine with dyno</td>
<td>7 hours</td>
</tr>
<tr>
<td>Generate and interpret dyno data</td>
<td>5 hours</td>
</tr>
<tr>
<td>Modify engine components</td>
<td>10 hours</td>
</tr>
<tr>
<td>Adjust to get best MPG</td>
<td>10 hours</td>
</tr>
<tr>
<td>Test adjustments</td>
<td>10 hours</td>
</tr>
<tr>
<td>Iterative adjustment and tests</td>
<td>10 hours</td>
</tr>
<tr>
<td><strong>Aerodynamics Work:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Aerodynamic Research:</strong></td>
<td></td>
</tr>
<tr>
<td>Research aerodynamic concepts and equations</td>
<td>10 hours</td>
</tr>
<tr>
<td>Determine how coefficient of drag and lift affect vehicle</td>
<td>8 hours</td>
</tr>
<tr>
<td>Design 2 shapes: models with interior / exterior wheels</td>
<td>6 hours</td>
</tr>
<tr>
<td>Draw 2 shapes in CAD</td>
<td>2 hours</td>
</tr>
<tr>
<td>Create 3D plots using Steelcase Printer</td>
<td>-</td>
</tr>
<tr>
<td>Create list of wind tunnel components</td>
<td>1 hour</td>
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<tr>
<td><strong>Aerodynamic Implementation:</strong></td>
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<tr>
<td>Build a wind tunnel test section</td>
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</tr>
<tr>
<td>Perform wind tunnel tests</td>
<td>8 hours</td>
</tr>
<tr>
<td>Generate and interpret wind tunnel data</td>
<td>2 hours</td>
</tr>
<tr>
<td>Make recommendation on body shape</td>
<td>-</td>
</tr>
<tr>
<td><strong>Materials Work:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Materials Research:</strong></td>
<td></td>
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<tr>
<td>First order calcs for chassis design</td>
<td>2 hours</td>
</tr>
<tr>
<td>Determine 3 material choices</td>
<td>8 hours</td>
</tr>
<tr>
<td>Research steering assemblies</td>
<td>7 hours</td>
</tr>
<tr>
<td>Research braking assemblies</td>
<td>7 hours</td>
</tr>
<tr>
<td>Determine frame design and material</td>
<td>8 hours</td>
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<tr>
<td><strong>Materials Implementation:</strong></td>
<td></td>
</tr>
<tr>
<td>Order of magnitude calcs for various materials</td>
<td>6 hours</td>
</tr>
<tr>
<td>Choose preferred chassis design</td>
<td>6 hours</td>
</tr>
<tr>
<td>Construct a CAD model of the vehicle</td>
<td>8 hours</td>
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<tr>
<td>Perform FEA analysis on key components</td>
<td>8 hours</td>
</tr>
<tr>
<td>Obtain materials</td>
<td>4 hours</td>
</tr>
<tr>
<td>Construct chassis</td>
<td>10 hours</td>
</tr>
<tr>
<td>Construct shell</td>
<td>10 hours</td>
</tr>
<tr>
<td>Construct steering assembly</td>
<td>5 hours</td>
</tr>
<tr>
<td>Construct braking assembly</td>
<td>5 hours</td>
</tr>
<tr>
<td><strong>Optimization Work:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Optimization Implementation:</strong></td>
<td></td>
</tr>
<tr>
<td>Test Vehicle</td>
<td>20 hours</td>
</tr>
<tr>
<td>Modify/Optimize vehicle based on testing</td>
<td>20 hours</td>
</tr>
<tr>
<td>Race in Competition</td>
<td>8 hours</td>
</tr>
<tr>
<td><strong>Documentation:</strong></td>
<td></td>
</tr>
<tr>
<td>Verbal presentation #1</td>
<td>1 hour</td>
</tr>
<tr>
<td>Verbal presentation #2</td>
<td>1 hour</td>
</tr>
<tr>
<td>Prepare PPFS</td>
<td>20 hours</td>
</tr>
<tr>
<td>PPFS Draft Due</td>
<td>-</td>
</tr>
<tr>
<td>Verbal presentation #3</td>
<td>1 hour</td>
</tr>
<tr>
<td>Verbal presentation #4</td>
<td>1 hour</td>
</tr>
<tr>
<td>Prepare final paper</td>
<td>20 hours</td>
</tr>
</tbody>
</table>
4 Design Process

4.1 Aerodynamics and Vehicle Envelope

The success of teams entering the Supermileage competition depends solely upon minimization of fuel consumption. A key design element in this minimization is the shape of the vehicle shell (coined “vehicle envelope”). One important dimension for such a design is that of aerodynamic performance. Team 6 therefore proposes to build two prototype body designs and test them in Calvin College’s wind tunnel with oversight from Professor DeJong. The following sections provide further detail in this process.

4.1.1 Alternatives

In the Pro/Engineer CAD software, Jacob designed two aerodynamic vehicle body types: Vehicle 1 has the wheels (not shown in figures) on the inside and Vehicle 2 on the outside. These models have been printed using a Fused Deposition Modeling (FDM) machine from Steelcase Inc. Vehicle 1 as seen in Figure 10 is longer than Vehicle 2 as seen in Figure 11 due to the wheels inside raising the shell height. With added height the vehicle needed to be longer in order to create a more steady flow path for the air.

![Vehicle 1 isometric view and dimensions](image1)

Figure 10: Vehicle 1 isometric view and dimensions

Notice that Vehicle 2 has a sharp cutoff in the back. Although fluid flow textbooks teach that these cutoffs produce turbulent flow in pipes, research shows that a Kammback rear actually improves efficiency in vehicles (“Kammback”). Kammback technology is often used on hybrids such as the Toyota Prius and the Honda Insight to increase fuel efficiency, thus it will be one of the options considered during wind tunnel testing.
As mentioned in the next section, the main goal of creating these models is for testing in the Calvin College wind tunnel, in order to determine which shell type will have the lowest coefficient of drag. The coefficient of drag is calculated using Equation 4, below.

\[
C_d = \frac{2F_d}{\rho \nu^2 A_{proj}}
\]

Eqn. 4

Where \( F_d \) is the drag force, \( \rho \) is the density of the air, and \( A_{proj} \) is the projected frontal area. This area will be calculated based on the dimensions of Jacob’s body, the wheel diameters (if they are inside the shell), and some factor that considers wiggle room needed for comfort and the aerodynamic shape. The constant \( C_1 \) is taken as the length of Jacob’s shoulders (23 inches). Adding 4 inches gives some extra room inside the cockpit:

\[
A_{proj} = C_1 D
\]

Eqn. 5

\[
C_1 = 23[in] + 4[in] = 27[in]
\]

Eqn. 6
The optimum tire size is discussed in Section 4.5.3 and was found to be 26 inches. Therefore the optimum projected frontal area will be slightly less than 620 square inches for a shell with the wheels on the inside.

4.1.2 Testing

The two models will be tested in the Calvin College wind tunnel. Professor DeJong created the wind tunnel in the summer of 2010 for use in his studies, education of fluid dynamics, and student projects. The wind tunnel can be used for wind speeds up to 200mph and is currently used for drag and sound testing. In order to measure the drag force, each vehicle will be mounted on twin cantilevers equipped with strain gages. The independent variable in this test will be the wind speed. With each wind speed setting, a reading from the strain output will be recorded. The resulting drag forces on test shapes will be modeled as one force at the end of a cantilever. Equation 7, below, shows the resulting force given any strain. Note that this drag force takes into account both form drag and pressure drag. A derivation can be found in Appendix A.

\[ F_D = \frac{E bh^2}{6L} \]  
Eqn. 7

The wind tunnel tests will accurately model our vehicle if it has the same Reynolds number as it would during actual operation. The Reynolds number can be calculated using Equation 8, below.

\[ Re = \frac{\nu L}{\mu} \]  
Eqn. 8

Where \( \rho \) represents the air density, \( \nu \) is the air speed, \( L \) is the characteristic length and \( \mu \) is the dynamic viscosity of the air. According to Prof. DeJong, the density of air does not change considerably until the airflow becomes turbulent. Therefore density is considered constant for ease of calculation. The kinematic viscosity will not change noticeably either and is taken as a constant. The characteristic length \( L \) is a geometric quantity relating to the vehicles overall length--this will change based on what scale the models are designed as. The air will move past the vehicle at a speed equal to the vehicle’s speed, therefore, in order to create the same Reynolds number in the wind tunnel as in real operation, the velocity of the air must be proportional to the characteristic length. The models will be created at a 1/12th scale--therefore the wind speed must be 12 times as fast in the wind tunnel to preserve the Reynolds number. Since the car will be operating at the lower speeds of 17mph or so, the models will be tested at the max wind speed of 200mph.

4.1.3 Final Selection

The primary goal of the wind tunnel tests is to help confirm design decisions regarding the overall vehicle shape. For example, two design alternatives are the following: wheels external to the vehicle envelope and wheels inside the vehicle shell. By creating models with these two variations, an objective decision will be made between these two options. Additionally, wind tunnel tests will provide insight to the changing aerodynamic drag at various velocities. Finally, the team hopes to investigate ground effects interacting between the ground and the underside of the vehicle.
4.2 Frame

The frame is an integral part of the vehicle's design; it must provide adequate structural integrity to support vehicle loads, provide the overall shape of the vehicle, and satisfy competition safety requirements. In addition, rolling and bearing resistances are proportional to the vehicle's weight, and thus it is advantageous to reduce the vehicle's total weight. For these reasons, material selection and a careful design are important to the overall success of the vehicle.

4.2.1 SAE Requirements

Competition requirements mandate a roll hoop that extends a minimum of 5 cm above the driver's helmet and is wider than the driver's shoulders as shown in Figure 12. The roll hoop must also withstand a 250 lbf force applied to it at any angle. The vehicle's structure must also include a fire wall that completely separates the driver from the engine. The fire wall must be made of steel or aluminum and a minimum of 0.032 inches thick; the fire wall may not have openings larger than 0.5 inches in diameter (SAE, 16-17).

![Figure 12: Roll Hoop Requirements](image)

4.2.2 Analysis

Team Supermileage is considering multiple vehicle envelopes as discussed in the aerodynamic section. The frame's final design cannot be completed until a final shape is selected. However, each vehicle shape being considered will have the same basic form as shown in Figure 13.
The frame team has developed a basic failure analysis of beam 1. The goal of this analysis is to guide the team’s selection of feasible materials for the rest of the frame. Figure 14 shows a simplified diagram of beam 1’s approximated loads. The beam is assumed to be simply supported at both ends and all of the loads are assumed to be static with the exception of the driver. The driver load is analyzed be an impact load and is calculated using Equation 9.

\[
F_{\text{driver,impact}} = \left[ 1 + \sqrt{1 + \frac{2nh}{\delta_{st}}} \right] F_{\text{driver,static}} \tag{Eqn. 9}
\]

Where \( \delta_{st} \) is the static deflection resulting from the weight of our driver, \( h \) is the height the load is falling from, and \( n \) is a correction factor. A height of \( \frac{1}{4} \) inches is assumed in the analysis—this accounts for the driver entering and exiting the vehicle as well as simulating the vehicle rolling over uneven roads. The impact force is inversely proportional to the deflection of the beam. Therefore increased material and section stiffness results in a smaller deflection and therefore a larger impact load—which is undesirable. The larger permitted deflection allows the material to absorb the energy of the impact over a longer period of time and therefore reduce the impact load. However, the deflection cannot be too large or the shape of the vehicle and the wheel alignment could be compromised. This analysis allows for a maximum deflection of about 1in (Norton 108-111).
Figure 14: Beam 1 loading diagram, all dimensions are in inches

Three different failure methods are considered for beam 1: beam in bending, holes in tear out, and bolts in shear. Considering the lifetime of this vehicle is under a year, fatigue will not be considered in this analysis. A minimum safety factor of 3 is being used for preliminary analysis—however the team hopes to refine the minimum safety requirement to 2 once all dimensions are finalized and Finite Element Analysis can provide a more accurate model of the system (Norton 18).

The maximum moment applied to the beam is directly under the driver. The stress due to bending is calculated with Equation 10, where \( c \) is the distance from the neutral axis, and \( I \) is the second moment of area (Norton 156).

\[
\sigma_{\text{bending}} = \frac{M_{\text{max}} c}{I} \quad \text{Eqn. 10}
\]

The beams will be assembled to other structural supports with bolts. The sides of the tubing must be thick enough to prevent the bolts tearing out of the members. The stress due to holes in bearing is calculated with Equation 11. Where \( R_B \) is the maximum shear force transferred from beam 1 to the rest of the body, \( n_{\text{bolts}} \) is the number of connecting bolts, and \( A_{\text{bearing}} \) is the projected area of the pin (Norton 154).

\[
\sigma_{\text{bearing}} = \frac{R_B}{n_{\text{bolts}} A_{\text{bearing}}} \quad \text{Eqn. 11}
\]
The bolts connecting the beams to the rest of the structure must be evaluated for shear failure. Shear stress in the bolts is calculated with Equation 12 where $A_{shear}$ is the cross sectional area of one bolt (Norton 162). The complete beam analysis can be seen in Appendix C.

$$\tau_{shear} = \frac{4}{3} \frac{R_B}{n_{bolts} A_{shear}}$$  \hspace{1cm} \text{Eqn. 12}

### 4.2.3 Alternatives

Team Supermileage is considering using wood, steel, and aluminum for structural support. The material properties of these materials are shown in Table 3.

Table 3: Considered materials for vehicle frame construction (Norton 988-993)

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m$^3$)</th>
<th>Yield Strength (MPa)</th>
<th>Modulus of Elasticity (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum (6061-T6)</td>
<td>2710</td>
<td>276</td>
<td>71.7</td>
</tr>
<tr>
<td>Aluminum (7075-T6)</td>
<td>2810</td>
<td>503</td>
<td>71.7</td>
</tr>
<tr>
<td>Steel (SAE 4340)</td>
<td>7870</td>
<td>1000</td>
<td>206.8</td>
</tr>
<tr>
<td>Wood (Douglas Fir)</td>
<td>550</td>
<td>56</td>
<td>13</td>
</tr>
</tbody>
</table>

Aluminum 6061-T6 is a common structural alloy that has good welding and workability properties. Aluminum 7075-T6 is high strength alloy that is commonly used in the aerospace applications; however, it is more difficult to weld. Steel 4340 is a common high strength alloy that has good weld and workability properties. Douglas fir is a common soft wood used to create lumber. (Norton 54-66).

Beam 1 is the longest and most heavily loaded beam in the vehicle and will therefore be used to guide material selection. The failure analysis described in the previous section was used to size beam one for commonly available shapes. Final beam selections can be seen in Table 4. Aluminum and steel beams are rectangular tubes as shown in Figure 15, and the wood is a rectangular cross-section.

Table 4: Summary of beam 1 sizing for considered material

<table>
<thead>
<tr>
<th>Material</th>
<th>Section Shape</th>
<th>Side A (in)</th>
<th>Side B (in)</th>
<th>Thickness C &amp; D (in)</th>
<th>Safety Factor</th>
<th>Weight (lb)</th>
<th>Deflection (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum (6061-T6)</td>
<td>Tube</td>
<td>2.85</td>
<td>1.024</td>
<td>0.067</td>
<td>3</td>
<td>3.66</td>
<td>0.35</td>
</tr>
<tr>
<td>Aluminum (7075-T6)</td>
<td>Tube</td>
<td>1.75</td>
<td>0.875</td>
<td>0.066</td>
<td>3.1</td>
<td>2.52</td>
<td>1.03</td>
</tr>
<tr>
<td>Steel (SAE 4340)</td>
<td>Tube</td>
<td>1.25</td>
<td>0.75</td>
<td>0.065</td>
<td>3.2</td>
<td>5.03</td>
<td>0.94</td>
</tr>
<tr>
<td>Wood (Douglas Fir)</td>
<td>Rectangle</td>
<td>3.8</td>
<td>1</td>
<td>NA</td>
<td>3</td>
<td>4.99</td>
<td>0.24</td>
</tr>
</tbody>
</table>
The total weight of beam 1 for each material is summarized in Table 4. This shows that aluminum 7075 T-6 results in the lightest beam, about 2.5 lbs.

4.2.4 Final Selection

A final frame design will be completed after a final vehicle shape is selected by the aerodynamic team. Preliminary calculations on beam 1 suggest that aluminum 7075 T-6 is the optimum material to reduce weight, however it is also very expensive and is not feasible unless it could be donated to the team. Aluminum 6061 T-6, douglas fir, and steel 4340 all provide adequate strength while maintaining a lightweight design and are available in Calvin’s workshops and are available to the team for free. Final material selection will depend on the specific beam and FEA results.

4.3 Engine

4.3.1 SAE Requirements

Each SAE Supermileage team is given a common engine, donated by the Briggs and Stratton Corporation. The 3.5hp base engine (Figure 16, below) is four-cycle, air cooled, carbureted, and equipped with side-mounted valves. Teams may modify the base engine within certain bounds. For example, the piston and crankcase may be machined or altered, but must still be identifiable as components of the base engine. In addition, a common fuel bottle (Figure 16) is provided to each team at the competition. Design of the engine and body must allow for the swift (45 seconds) removal and replacement of the fuel bottle so that competition organizers can easily measure fuel usage at the competition.
4.3.2 Testing

Internal combustion engines have been the subject of over a century of research and implementation, both by academia and the automotive industry. Engine efficiency is often a function of many interdependent features and components; theoretical analysis can be very complicated and unreliable. For this reason, the team decided to construct an engine dynamometer. With the ability to empirically test the engine, the team would be able to objectively quantify engine improvements. Additionally, the team could discover optimum engine operation parameters, e.g. the rpm at which the engine performs most efficiently.

Two dynamometer options exist at the time of this report. First, a local business, Baker Engineering Inc. from Nunica, MI, was receptive to the idea of partnering with the team in testing and modifying the engine. They have professional-quality small engine dynamometer test equipment. Baker Engineering has years of experience designing and building performance gas engines, both large and small. Depending on the availability of their engineers, Calvin Supermileage hopes to make used of the Baker Engineering’s advice and equipment.

The second option is for Calvin Supermileage to build dynamometer on campus. The proposed dynamometer is an engine dyno, meaning that it is connected to the output shaft of the engine itself, as opposed to a chassis dyno which takes measurements at the wheels of the vehicle. Modeled after a design similar to that of Figure 17, the Calvin dynamometer will couple the petrol engine to the DC motor. The power would then be dissipated through resistance wires powered by the DC motor. Because the motor is mounted on non-fixed trunnions, a fixed lever-arm mounted to the DC motor will give the torque acting on the motor by applying a force to the load cell.
A tachometer will measure the angular velocity of the output shaft. The engine power then, is given by Equation 13, when $\omega$ is in radians per second.

$$Power = T\omega$$  \hspace{1cm} \text{Eqn. 13}

From this equation, the peak engine power can be found as a function of rpm. To find peak efficiency, the rate at which fuel is consumed must also be measured. The addition of a fluid flow-meter giving the fuel consumption rate can give the effective fuel-volume efficiency (Joules/mL), calculated by Equation 14.

$$EFE = \frac{\text{power}}{V_{\text{fuel}}}$$  \hspace{1cm} \text{Eqn. 14}

By building a dynamometer, Calvin Supermileage will have the ability to quantify any modifications made to the engine. This will allow the team to definitively know if a modification has improved the efficiency of the engine. By running the engine at different rpms, the team will determine most efficient engine speed. This information can then be used when designing the gear ratios in the transmission and power train.

As of the writing of this report, a dynamometer has not been constructed; the team is in possession of a DC motor, but it is rated at 2.5 hp, and will not provide accurate results across the entire power band of the 3.5 hp stock B&S motor. The team is in contact with several local businesses that may be willing to donate an appropriate DC motor to the project.

As of the writing of this report, the donated competition motor had not arrived from Briggs and Stratton. As such, no testing or modifications had been performed.

4.3.3 Alternatives

Of the hundreds of ways the engine could be modified, the team chose just three to consider. Many other options exist, but time and budget constraints forced the team to limit the scope of possible engine
modifications to those that initially seemed most feasible, effective and affordable. Figure 18 shows the options under consideration.

**Figure 18: Modification options for the petrol engine**

**Compression Ratio and “Running Lean”**

Without any major design modifications to the engine itself, certain parameters can be adjusted to increase fuel efficiency. With fuel delivery executed by the carburetor, adjustments can be made to the air-to-fuel mass ratio or AFR, found in Figure 19.

$$AFR = \frac{m_{\text{air}}}{m_{\text{fuel}}}$$  \hspace{1cm} \text{Eqn. 15}

When the AFR matches the molecular ratio for a complete combustion of the fuel, it is called the stoichiometric AFR, and is about 14.7:1 for gasoline (Stevens). Typically, a richer AFR (more fuel and a smaller AFR) can produce more power, while a leaner AFR can product more efficiency. The AFR for best efficiency varies from engine to engine and is a function of many design variables. Modern cars have found peak efficiency anywhere from 15.7:1 to 17.6:1 (Stevens, and Edgar). The team, using the dynamometer, can adjust the carburetor to test various AFRs and establish the best ratio for fuel efficiency.

Another engine modification under consideration is the compression ratio. The compression ratio is the ratio of the cylinder volume at the bottom of the stroke (bottom dead center, or TDC) to the volume at the top of the stroke (TDC), Equation 16 (Cengel, 361).

$$CR = \frac{V_{BDC}}{V_{TDC}}$$  \hspace{1cm} \text{Eqn. 16}

A higher compression ratio results in greater pressures during the combustion phase of the engine cycle. The CR could be increased by adding material (through welding) to the combustion chamber, thus reducing the $V_{TDC}$. These higher pressures can in turn result in greater energy release from the combustion cycle. In an ideal engine cycle (Otto cycle) with air modeled as an ideal gas, the efficiency of the engine is given by Equation 17 (Cengel, 361). Here, $k$ is a constant specific heat ratio for gasoline equal to 1.4.
$\eta_{ideal} = 1 - \frac{1}{CR^{\kappa-1}}$  

Eqn. 17

Even though no engine can perform as thermodynamically ideal, it is interesting to see the increase in efficiency as a function of compression ratio. The base engine provided by Briggs and Stratton has a compression ratio of 6:1 (Briggs and Stratton FAQ); increases up from 6 are shown in Figure 19.

![Efficiency Increase with Higher Compression Ratios](image.png)

Figure 19: Increasing compression ratio provides increased engine efficiency

There are several drawbacks to increasing the compression ratio. First, an increase in CR drives an increase in cylinder pressure during the engine cycle. The materials and construction of the engine head and cylinders might not withstand the pressure increases. Second, with the pressure increases come temperature increases. At higher temperatures, engine components see greater stresses and could melt. In addition, if the fuel reaches its auto-ignition temperature before the spark plug fires, a spontaneous combustion could take place creating engine knock. Figure 20 shows the nominal ignition temperature (Transportation Energy Data, 1) for the competition fuel (iso-octane) in relation to the increased temperatures.

In a thermodynamically ideal engine model, any compression ratio greater than 8 could produce temperatures above the auto ignition temperature for iso-octane. Several things, however, will likely allow for these high temperatures. First, the actual engine is not thermodynamically ideal, and therefore these will be heat loss during the compression cycle, and the temperature will not increase as dramatically as shown in these calculations. Second, the nominal ignition temperature is not measured in conditions at all similar to the inside of an engine, and may in fact be several hundred degrees higher than nominal (Smyth and Bryner 247). Because of this, it is not likely that engine knock will be a problem.
The stock 3.5hp Briggs and Stratton comes with a fixed jet carburetor. Essentially, carburetors deliver fuel to the combustion chamber as a function of the flow-rate of air into the cylinder. The intake air passes through a venturi “throat” in the carburetor. The increase in air velocity creates a pressure drop which draws in liquid fuel through a small orifice in the sidewall. The flow-rate of the air (controlled by the throttle) then determines the amount of fuel drawn into the combustion stream. The carburetor can be adjusted (as suggested above) to different proportions of fuel and air (AFR, air-fuel ratio). A basic diagram of a carburetor is found in Figure 21.
Fuel injection is another technology for delivering fuel to the combustion stream, but does so quite differently from carburation. Whereas the carburetor draws liquid fuel into the air with a venturi pressure drop, a fuel injector uses tiny jets to atomized the fuel and spray it into the combustion stream based on parameters such as air flow rate, engine speed, throttle, temperature and others (Stone, 122). The parameters are measured with sensors and processed by an engine control unit, or ECU. The ECU determines the amount of fuel to inject based on algorithms programmed into the ECU. The flow diagram showing a fuel injection system is shown in Figure 22.

The injector can either be located before the intake valve (port injection) or as part of the combustion chamber itself (direction injection). In either case, atomized fuel is mixed with air for combustion in the cylinder, Figure 23.
Figure 23: Left, a typical fuel injector sprays atomized fuel
(http://upload.wikimedia.org/wikipedia/commons/2/29/Injector3.gif). Middle: direct injection sprays fuel into the cylinder. Right, port injection sprays fuel upstream of the intake valve (Walton, 99)

For maximum efficiency, fuel injection has several advantages over traditional carbureted engines. First, the fuel is atomized when ignition occurs versus a liquid fuel mixture in carburation. This atomized fuel burns more efficiently than liquid fuel and results in greater engine efficiency (Walton, 98). Second, fuel injection is much more responsive to the operating environment; the ECU can vary the amount of fuel based on sensor data and react instantly to changes in temperature and pressure. This means less wasted fuel, and better AFR at all operating speeds. Third, there is no venturi “throat” that constricts the intake flow which results in easier engine “breathing” and less flow-power loss.

Unfortunately, converting a carbureted engine to one with fuel injection is no easy task. A host of new engine components must be affixed to the engine: sensors, injections, pumps, an ECU, etc. It will be a challenge merely to get all these new components to work together to that the engine operates at all; it is another challenge to optimize the operation so that the engine runs more efficiently. Figure 24 shows some pros and cons of fuel injection fuel delivery system.
Figure 24: Fuel injection is costly and complicated, but more efficient than carburation.
(Stone, 121. Walton, 95-99, Pulkrebek, 179-181)

Without the advice of engineers at Baker Engineering Inc., it is unlikely that Calvin Supermileage will be able to convert the engine to run with fuel injection. The increased costs and design difficulties are likely too great to overcome within the budget and timing of the senior design project.

**Combustion Chamber and Head**

The Briggs and Stratton base engine has a side-valve arrangement, similar to the one shown in Figure 25. In this arrangement, the valves and spark plug are offset from the center of the piston. The theoretical fluid dynamics involved in the flow and combustion are very complicated and difficult to perform accurately. Despite this, there are some generalizations than can be made about combustion chamber design.

Side valve configurations have an upper limit of 6:1 for the allowable compression ratio. This is because the explosion trajectory is not in line with the piston motion, and the zero-clearance “squish” region on one side of the piston can cause pockets of high pressure resulting in engine-knock (Stone, 100). In addition, the flow of the air-fuel mixture can create poor burn profiles in a side-valve configuration. Many other designs exist, each with a particular design benefits. Figure 26 shows several common combustion chamber designs.

For maximum efficiency, it is important to ensure an even fuel mixture throughout the chamber, minimize flow-power losses through valve and port design, and low-mechanical wear (Stone, 100. Pulkrebek, 249, Second Chance Garage. Morgan.). With the cooperation of a local machine shop, Calvin Supermileage
would be able to design a new combustion chamber with an overhead valve configuration. Additional design would be needed to arrange the valve push-rods. Both designs (a) and (b) could be implemented with the need for additional piston modification. Head modifications are quite common among performance enthusiasts, and so the feasibility is quite high for this engine modification.

Figure 25: Base engine side-valve combustion chamber, offset from cylinder

Figure 26: Combustion chamber designs: (a) wedge chamber, (b) hemispherical head, (c) bowl in piston, and (d) bath-tub head (Stone, 104).
4.3.4 Recommended Modifications

Calvin Supermileage has examined several options for engine modification, and determined the need for dynamometer testing. Based on ease of fabrication, cost, and predicted efficiency improvements, the team will likely modify the compression ratio and adjust the AFR with the carburetor. Fuel injection and head design are possible, but the team will seek the advice of local professionals before attempting those options. The proposed plan is listed below in order of probability of execution (1 being most probable).

**Testing:**
1. Baker Engineering dynamometer
2. Calvin Supermileage home-made dyno

**Engine Mods**
1. Adjust carburetor to run lean AFR ~ 16:1
2. Increase compression ratio by welding in the combustion chamber
3. Redesign combustion chamber to wedge or bath-tub design
4. Implement a fuel injection system

4.4 Steering

4.4.1 SAE Requirements

The steering mechanism is an important characteristic of the vehicle, which must both enable safe and effective maneuvering of the car, and satisfy the specifications set out by SAE. The track that the vehicle will compete on is a 1.6-mile oval test track shown in Figure 27, below.

![Test track for Supermileage® competition](image)

Figure 27: Test track for Supermileage® competition
This in itself doesn’t require much of a turning radius, however the vehicle must also pass a maneuverability course before it is allowed to compete. This course, as illustrated in Figure 28, consists of completing an untimed 180 degree turn, and then weaving in and out of 4 cones placed 25 feet apart in under 15 seconds.

![Maneuverability Course Diagram](image)

Figure 28: Maneuverability course

In order to successfully complete this course then, the vehicle must have a maximum turning radius of 50 ft. Another requirement by SAE is that the steering must have a “natural” response. In other words, if the driver turns the steering wheel left, the car will go left, and vice-versa.

### 4.4.2 Alternatives

As the basic components of the Supermileage vehicle are quite similar to a go-kart, go-kart steering systems were studied and analyzed. There are two main designs for go-kart steering systems: Wagon Style systems, and steering knuckle systems.

**Wagon Style System:**

The first type of system is the most classic, but also the most elementary. This simple system simply consists of two wheels permanently fixed to an axle, which is then rotated by the steering column. See Figure 29 below.
**Steering Knuckle System:**

This system is more complicated; however it also solves many of the problems of the Wagon wheel type. The steering knuckle system consists of wheels mounted to “knuckles”, which are in turn mounted to an axle. These knuckles are then driven by a 4-bar linkage connected to the steering column, as shown in Figure 30. Because it is driven by a 4-bar linkage, this system is much more stable. Another advantage of being driven by a 4-bar linkage is that by adjusting the lengths of various bars, optimizations and changes can be implemented (Steering Systems).
4.4.3 System Selection

Because of its many advantages, the steering knuckle system will be employed in our Supermileage vehicle. The final product will be similar to Figure 31.

Figure 31: A knuckle steering system implemented in vehicle

4.5 Rolling Resistance

Rolling resistance is a major parasitic drag that occurs between the tires and the ground. For a cyclist, rolling resistance can account for up to 80% of drag at speeds of 6mph, and as much as 20% at speeds of 25mph, therefore decreasing rolling resistance will make the Supermileage vehicle much more efficient. When a tire holding up weight bulges against the ground it increases the contact area, see Figure 32. This area is directly related to the amount of friction the tire feels from the ground. To minimize the friction one needs to decrease this contact patch by investing in a thinner tire or increasing the tire pressure in the current tire.
There will be no testing on different tires due to the high upfront costs of buying tires and lack of a testing fixture similar to Figure 33. Typically, tires are tested on a rolling drum while a known weight is pressed down on the tire. The drum rolls at a known speed and the torque required to keep the drum rolling equates to a rolling resistance coefficient.

To simplify, the team will consider the power to propel the vehicle forwards as a function of the rolling resistance, weight and velocity. Thus with just a researched rolling resistance the team can determine the power to overcome this drag using Equation 18, below.

\[ P_{rr} = C_{rr}, Nv \]  

Eqn. 18
Where $C_r$ is the rolling resistance coefficient, $N$ is the weight on the tire and $v$ is the velocity of the vehicle. Since the Supermileage vehicle is much heavier than a road bike, it is very important to note the weight in these calculations.

### 4.5.2 Alternatives

Among bike tire enthusiasts, there is some rolling resistance data—but it is not common for a bike tire company to publish these numbers as they are dependent on too many factors: weight, tire pressure, temperature, speed...etc. However, as one can see in Table 5, rolling resistance coefficients for high-end tires are quite small, but when multiplied by weight and velocity the magnitude of the power loss due to the rolling resistance can be upwards of 70 watts.

<table>
<thead>
<tr>
<th>Tires</th>
<th>$C_{rr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Veloflex Carbon</td>
<td>0.0049</td>
</tr>
<tr>
<td>Gommitalia Platinum</td>
<td>0.0053</td>
</tr>
<tr>
<td>Vittoria Corsa Evo CX</td>
<td>0.0054</td>
</tr>
<tr>
<td>Vittoria Corsa Evo KX</td>
<td>0.0057</td>
</tr>
<tr>
<td>Continental Competition</td>
<td>0.0059</td>
</tr>
</tbody>
</table>

There seems to be inconclusive data online as to whether a larger or smaller tire has less rolling resistance. However, most sites seem to indicate that cyclists choose a larger diameter tire for its lower rolling resistance. As said before, the rolling resistance is a function of several variables. To simplify, the analysis will only consider the diameter. Equation 19 relates the coefficient of rolling resistance to the diameter $D$ of the tire and the amount of vertical deflection $z$.

$$C_{rr} = \sqrt[12]{\frac{z}{D}}$$

Eqn. 19

Using this equation, tires from Table 5 have a vertical deflection of about 700-1000 micro-inches. Note that this analysis assumes a constant deflection amongst tire options. Using the constant deflection assumption and a 26-inch tire, the coefficient of rolling resistance becomes 0.0057 for a high-end tire.

### 4.5.3 Final Selection

There are hundreds of bike tires to choose from with differing coefficients of rolling resistance, price and size. The team will make its final choice on tire based on size first, then price and rolling resistance. The order of magnitude calculations discussed in section 2.4 was used to determine which tire diameter would optimize the aerodynamic drag and rolling drag. Figure 34, below, shows the power loss (in watts) of the combined drag and rolling resistances as a function of the tire diameter. An optimum diameter was found using the following equations for power loss in the aerodynamic drag and rolling resistance.

$$P_{drag} = 0.5\rho C_1 DC_d v^3$$

Eqn. 20
By taking the derivative of $P_{\text{resist}}$ with respect to $D$ and setting it equal to zero, a minimum was determined. The optimum tire diameter becomes:

$$D_{\text{opt}} = \frac{\sqrt{zW}}{C_\gamma C_d v^2}^{2/3}$$

Eqn. 23

Plotting this for a few velocities between the required 15-25mph yields Figure 34. For example, the optimal tire diameter that minimizes the power losses due to aerodynamic drag and the rolling resistance for the vehicle traveling at 20mph is about 24 inches. The team already owns a 26-inch tire frame with high efficiency bearings. To save on cost the team decided on a 26-inch tire instead of a potential 28-inch tire such as those in Table 5, above.
5 Vehicle Costs

5.1 Cost Estimation Development

5.1.1 Registration Costs

The registration costs for the SAE Supermileage® competition are unfortunately not cheap. At $600 for registration, plus $100 for SAE memberships, registration costs run over half of our ideal budget.

Table 6: Registration costs

<table>
<thead>
<tr>
<th>Registration Costs:</th>
<th>Quantity</th>
<th>Unit Cost</th>
<th>Total Cost</th>
<th>Possibility of pilfering from last year:</th>
<th>Possibility of donation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Registration Fee:</td>
<td>1</td>
<td>$600</td>
<td>$600</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SAE Team Membership:</td>
<td>5</td>
<td>$20</td>
<td>$100</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total Cost:</td>
<td></td>
<td></td>
<td>$700</td>
<td>$700</td>
<td>$700</td>
</tr>
</tbody>
</table>

5.1.2 Body and Frame Costs

The main expense in this category is, as might be expected, the aluminum material. This will be a key component to try to get donated then. Most of the other costs in the frame come from various specifications from SAE regarding the competition. For example, all vehicles must be equipped with a 3-point restraint seat-belt, and the vehicle must be able to be stopped relatively quickly, which requires disk-brakes.

Table 7: Body and frame costs

<table>
<thead>
<tr>
<th>Body/Frame Costs:</th>
<th>Quantity</th>
<th>Unit Cost</th>
<th>Total Cost</th>
<th>Possibility of pilfering from last year:</th>
<th>Possibility of donation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk Brakes:</td>
<td>2</td>
<td>$20</td>
<td>$40</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Bike Tubes:</td>
<td>3</td>
<td>$5</td>
<td>$15</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Bike Tires:</td>
<td>3</td>
<td>$20</td>
<td>$60</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Bike Rims:</td>
<td>3</td>
<td>$20</td>
<td>$60</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Aluminum Tubing (per foot):</td>
<td>40</td>
<td>$5</td>
<td>$200</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Aluminum Firewall:</td>
<td>1</td>
<td>$14</td>
<td>$14</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Tie Rods:</td>
<td>4</td>
<td>$10</td>
<td>$40</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Seat Material:</td>
<td>1</td>
<td>$15</td>
<td>$15</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>3-Point Seat Belt:</td>
<td>1</td>
<td>$40</td>
<td>$40</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Total Cost:</td>
<td></td>
<td></td>
<td>$484</td>
<td>$214</td>
<td>$95</td>
</tr>
</tbody>
</table>
5.1.3 Electrical Costs

Most of the electrical costs are driven by SAE requirements also. The requirements specify that the vehicle needs brake lights, for instance. This then necessitates the use of wiring, a battery, and some lights.

Table 8: Electrical costs

<table>
<thead>
<tr>
<th></th>
<th>Quantity</th>
<th>Unit Cost</th>
<th>Total Cost</th>
<th>Possibility of pilfering from last year:</th>
<th>Possibility of donation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric Starter:</td>
<td>1</td>
<td>$35</td>
<td>$35</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Battery (12V):</td>
<td>1</td>
<td>$30</td>
<td>$30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Brake Lights and Wiring:</td>
<td>1</td>
<td>$30</td>
<td>$30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Fuse Box:</td>
<td>1</td>
<td>$20</td>
<td>$20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Kill Switches:</td>
<td>1</td>
<td>$30</td>
<td>$30</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Speedometer:</td>
<td>1</td>
<td>$15</td>
<td>$15</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total Cost:</strong></td>
<td></td>
<td></td>
<td><strong>$160</strong></td>
<td><strong>$115</strong></td>
<td><strong>$160</strong></td>
</tr>
</tbody>
</table>

5.1.4 Engine and Powertrain Costs

Fortunately, Briggs & Stratton graciously agrees to donate a 3.5 horsepower engine to all the teams registered for the competition. This saves us a ton of money, by eliminating a potentially extremely expensive component. The other components are necessary for the power train, but last year’s prototype vehicle will be inspected to see if anything can be salvaged, in order to cut costs.

Table 9: Engine and powertrain costs

<table>
<thead>
<tr>
<th>Engine/Powetrain Costs:</th>
<th>Quantity</th>
<th>Unit Cost</th>
<th>Total Cost</th>
<th>Possibility of pilfering from last year:</th>
<th>Possibility of donation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine:</td>
<td>1</td>
<td><em>donated</em></td>
<td><em>donated</em></td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Clutch:</td>
<td>1</td>
<td>$60</td>
<td>$60</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sprockets:</td>
<td>1</td>
<td>$50</td>
<td>$50</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chains:</td>
<td>2</td>
<td>$20</td>
<td>$40</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bearings:</td>
<td>3</td>
<td>$20</td>
<td>$60</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total Cost:</strong></td>
<td></td>
<td></td>
<td><strong>$210</strong></td>
<td><strong>$150</strong></td>
<td><strong>$210</strong></td>
</tr>
</tbody>
</table>

5.1.5 Shell Costs

The shell will most likely be a huge section of the budget. As the frame will be different from last year’s vehicle, there is no possibility of re-using material. This will mean that an entire new shell of chicken-
wire and fiberglass cloth will need to be constructed; the price to do this could be high. However, the
team is also following up the possibility of getting it all donated.

Table 10: Shell costs

<table>
<thead>
<tr>
<th>Shell Costs:</th>
<th>Quantity</th>
<th>Unit Cost</th>
<th>Total Cost</th>
<th>Possibility of pilfering from last year:</th>
<th>Possibility of donation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicken Wire:</td>
<td>1</td>
<td>$16</td>
<td>$16</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Foam:</td>
<td>10</td>
<td>$4</td>
<td>$40</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Fiberglas Cloth:</td>
<td>1</td>
<td>$100</td>
<td>$100</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>FB Resin:</td>
<td>2</td>
<td>$30</td>
<td>$60</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>FB Hardener:</td>
<td>2</td>
<td>$16</td>
<td>$32</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Acrylic Windshield:</td>
<td>2</td>
<td>$20</td>
<td>$40</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1st Stage Paint:</td>
<td>10</td>
<td>$15</td>
<td>$150</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Primer:</td>
<td>10</td>
<td>$4</td>
<td>$40</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Gloss:</td>
<td>10</td>
<td>$4</td>
<td>$40</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Sandpaper:</td>
<td>7</td>
<td>$6</td>
<td>$42</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>Total Cost:</strong></td>
<td></td>
<td></td>
<td>$560</td>
<td>$560</td>
<td>$0</td>
</tr>
</tbody>
</table>

5.1.6 Total Costs

As can be seen in the tables below, it is estimated that the base cost of constructing this vehicle will be over $2100. However, donations are actively being pursued, and with the possibility of borrowing some materials from last year’s vehicle, it is believed that we can get this cost down to $1200.

Table 11: Total costs

<table>
<thead>
<tr>
<th>Total Costs:</th>
<th>Cost with heavily pilfering from last year:</th>
<th>Cost with getting substantial donations:</th>
<th>Cost with donations and pilfering:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Cost (nothing borrowed or donated):</td>
<td>$2,114</td>
<td>$1,739</td>
<td>$965</td>
</tr>
</tbody>
</table>

Because this vehicle is a one-time project, it is a little difficult to have a “production cost.” For the purposes of this project, production costs were assumed to take into account the cost of designing, constructing, and optimizing the vehicle. This cost would then be exactly the same as shown above, with the addition of design and construction time. It is estimated from our Work Breakdown Schedule, that this project will require approximately 320 hours per person. This project is being worked on by a team of 5. Therefore if we assume that each team member is being paid $25/hour, the total cost of this project is $125/hr. At this rate, the total cost of labor/design/construction works out to be $40,000. If we assume a worst-case scenario that nothing can be either borrowed or donated, the production cost estimate becomes $42,114.
6 Conclusion

The above chapters substantiate the feasibility for Team 6 to construct a vehicle to compete in the SAE Supermileage® competition. Specific design solutions outlined above provide insight into the following vehicle categories: aerodynamics, engine optimization, frame design, engine design, and power transmission. These design solutions were selected in effort to minimize vehicle inefficiencies while also maintaining the SAE Supermileage® rules. In addition to achieving success in this design, Team 6 also hopes to continue learning team skills. Looking forward, several milestones still lie ahead. A few examples include vehicle fabrication, engine testing, and wind tunnel testing. A schedule containing all critical milestones will be created for interim and Spring Semester. Amidst the long road ahead, Team 6 determines to assess and overcome the many challenges that are bound to arise along the way.
7 Acknowledgements

Many people offered technical assistance and gave advice toward this project. We would like to especially thank the following people:

   Professor Ned Nielson
   Professor Richard Dejong
   Professor Matthew Heun
   Phil Jasperse
   Briggs and Stratton
   Ren Tubergen

In addition, we would also like to thank our families and friends for giving us support and encouragement in undertaking this project.
8 References

   <http://www.sae.org/domains/students/competitions/supermileage/>.


Engineering Scratch Drive. <S:\Engineering\Scratch\ENGR 339\SAE Supermileage SDP>.


9 Appendices

9.1 Appendix A: Wind Tunnel Stress Calculations

Derivation for Equation 7:

\[ \sigma = \frac{Mc}{I} \]

\[ I = \frac{bh^3}{12} \]

\[ M = FL \]

\[ \therefore \sigma = \frac{6F_D L}{bh^2} \]

\[ \sigma = \varepsilon E \]

\[ \therefore F_D = \frac{E \varepsilon bh^2}{6L} \]

\[ F_D = C_D A \frac{\rho V^2}{2} \]

\[ \therefore C_D = \frac{E \varepsilon bh^2}{3L \rho AV^2} \]
9.2 Appendix B: EES Order of Magnitude Calculations

Team 6

Order of Magnitude Calculations

November 4

$V_{max} = 6.706 \text{ m/s}$
$V_{max} = 11.175 \text{ ft/s}$

Design Variables

$V = 6.706 \text{ [m/s]}$

Vehicle Mass

$M_{vehicle} = 52.2 \text{ [kg]}$
$M_{driver} = 51.2 \text{ [kg]}$
$M_{combined} = M_{vehicle} + M_{driver}$

Wheel Diameter $= 0.508 \text{ [m]}$

Vehicle's Projected Area

Area $= 0.75 \text{ [m}^2\text{]}$

Coefficient of Drag

$C_d = 0.12$

Rolling Resistance Coefficient

$C_r = 0.0055$

Coefficient of Bearing Friction

$C_b = 0.0015$

Constants

$g = 9.807 \text{ [m/s}^2\text{]}$
$\rho = 1.2 \text{ [kg/m}^3\text{]}$

Force Calculations

Force Drag

$F_{drag} = C_d \cdot \text{Area} \cdot \frac{V^2}{2} \cdot \rho$

Power $= F_{drag} \cdot V$

Force Rolling Resistance
\[ F_N = M_{\text{combined}} \cdot \theta \]
\[ F_\pi = C_\pi \cdot F_N \]
\[ \text{Power}_{\text{rolling resistance}} = F_\pi \cdot V \]

**Force Bearing Resistance**
\[ F_\delta = C_\delta \cdot F_N \]
\[ \text{Power}_{\text{Bearing resistance}} = F_\delta \cdot V \]

**Sum of Power Loss**
\[ \text{Sum}_{\text{Power Loss}} = \text{Power}_{\text{drag}} + \text{Power}_{\text{rolling resistance}} + \text{Power}_{\text{Bearing resistance}} \]

**Basic Efficiency calc's**

*lower heating value of gasoline*
\[ \text{LHV} = 4.44 \times 10^7 \text{ [J/kg]} \]

**Engine efficiency**
\[ \text{Engine}_{\text{eff,nom}} = 0.165 \]
\[ V_{\text{i}} = V \cdot 1 \text{ [m]} \]
\[ \text{energy}_{\text{eff,compression factor}} = -0.01748 \cdot V_{\text{i}} + 1.2079 \cdot V_{\text{i}}^2 + 13.513 \cdot V_{\text{i}} \]
\[ \text{energy}_{\text{eff}} = \text{energy}_{\text{eff,nom}} \cdot \text{energy}_{\text{eff,compression factor}} \]

**Engine_{eff} = Engine_{eff,nom} \cdot energy_{eff,compression factor}**

**Minimum power required to make up for losses**
\[ P_{\text{Engine,req}} = \frac{\text{Sum}_{\text{Power Loss}}}{\text{Engine}_{\text{eff}}} \]

**mass and volumetric flow rate**
\[ \dot{m}_{\text{fuel,req}} = \frac{P_{\text{Engine,req}}}{\text{LHV}} \]
\[ S_{\text{fuel}} = 720 \text{ [kg/m}^3\text{]} \]
\[ \dot{V}_{\text{fuel,req}} = \frac{\dot{m}_{\text{fuel,req}}}{S_{\text{fuel}}} \]
\[ \dot{V}_{\text{fuel,req,gal}} = \dot{V}_{\text{fuel,req}} \cdot \frac{264.2 \text{ gal}}{\text{m}^3} \]
\[ \text{Estimated}_{\text{Fuel,ton}} = \frac{V}{\dot{V}_{\text{fuel,req,gal}}} \cdot \frac{0.000621371 \text{ mile}}{\text{m}} \]
9.3 Appendix C: MathCAD Frame Calculations

Beam 1: Main Beams
Given:
Pinned Ends
total length = 73 in
length between supports = 64 in

Assumptions:
Most likely failure points are beam in bending, holes in bearing (tear out), bolts in shear. Lifetime of less than 1 year, thus fatigue will not be considered.

Beam Properties and Calculated Reactions:
Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Strength</th>
<th>Modulus of Elasticity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 6061-T6</td>
<td>270 MPa</td>
<td>71.7 GPa</td>
<td>2710 kg/m³</td>
</tr>
<tr>
<td>Aluminum 7075-T6</td>
<td>503 MPa</td>
<td>71.7 GPa</td>
<td>2810 kg/m³</td>
</tr>
<tr>
<td>Steel (SAE 4340)</td>
<td>1000 MPa</td>
<td>206.8 GPa</td>
<td>7840 kg/m³</td>
</tr>
<tr>
<td>Wood Doug Fir</td>
<td>44 MPa</td>
<td>13 GPa</td>
<td>498 kg/m³</td>
</tr>
</tbody>
</table>

Section Properties
Aluminum Rectangular Tube

<table>
<thead>
<tr>
<th>Material</th>
<th>A_{AE,r}</th>
<th>B_{AE,r}</th>
<th>C_{AE,r}</th>
<th>D_{AE,r}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 6061</td>
<td>2.85 in</td>
<td>1.024 in</td>
<td>0.067 in</td>
<td>0.067 in</td>
</tr>
<tr>
<td>Aluminum 7075</td>
<td>1.75 in</td>
<td>0.875 in</td>
<td>0.066 in</td>
<td>0.066 in</td>
</tr>
</tbody>
</table>

\[
I_{AE,r} = \frac{\left( B_{AE,r} A_{AE,r}^3 \right)}{12} - \frac{\left[ (B_{AE,r} - 2C_{AE,r}) (A_{AE,r} - 2D_{AE,r})^3 \right]}{12} = 0.489 \text{ in}^4 
\]

\[
c_{AE,r} = \frac{1}{2} A_{AE,r} = 1.425 \text{ in}
\]

\[
I_{AE,7} = \frac{\left( B_{AE,7} A_{AE,7}^3 \right)}{12} - \frac{\left[ (B_{AE,7} - 2C_{AE,7}) (A_{AE,7} - 2D_{AE,7})^3 \right]}{12} = 0.129 \text{ in}^4 
\]

\[
c_{AE,7} = \frac{1}{2} A_{AE,7} = 0.875 \text{ in}
\]
Steel 4340

\[ I_{SE} := \left[ \frac{BSE \cdot A_{SE}^3}{12} \right] - \left[ \frac{(BSE - 2CSE)(A_{SE} - 2D_{SE})^3}{12} \right] = 0.049 \text{ in}^4 \]

\[ c_{SE} := \frac{1}{2} \cdot A_{SE} = 0.625 \text{ in} \]

**Wood Rectangular**

\[ b_w := 1 \text{ in} \]
\[ h_w := 3.8 \text{ in} \]
\[ I_w = \frac{b_w \cdot h_w^3}{12} = 4.573 \text{ in}^4 \]
\[ c_w := \frac{1}{2} \cdot h_w = 1.9 \text{ in} \]

**Beam Specifications:**

\[ L_1 := 64 \text{ in} \quad \text{length between supports} \]
\[ L_2 := 73 \text{ in} \quad \text{total length of beam} \]

**Loads**: Driver, Roll bar, Engine, Shell, Self Weight.

**Point Loads**

\[ F_{roll} := \frac{50}{2} \text{ lbf} = 111.2 \text{ N} \quad x_{roll} := 41 \text{ in} \]
\[ F_{engine} := \frac{30}{2} \text{ lbf} = 66.723 \text{ N} \quad x_{engine} := 50 \text{ in} \]
\[ n_{shell} := 8 \quad \text{number of shell connections} \]
\[ x_{shell_1} := 0 \text{ in} \]
\[ x_{shell_2} := 27 \text{ in} \]
\[ x_{shell_3} := x_{roll} = 41 \text{ in} \]
\[ x_{shell_4} := 64 \text{ in} \]

**Impact Loads: Driver**

Norton P. 108-111

Energy Method

\[ P_{driver} := \frac{135}{2} \text{ lbf} = 300.3 \text{ N} \quad x_{driver} = 36 \text{ in} \quad b_{driver} := (L_1 - x_{driver}) \]

\[ h := \frac{1}{4} \text{ in} \quad \text{correction factor} \quad n_{def} := 1 \]

**Uniformly Distributed Loads**

\[ W_{vehicle\_self\_weight} := \frac{70 \text{ lbf}}{2L} = 0.547 \text{ lbf/in} \]
1 Selected Material: 6061
Selected Shape: Tube

\[
\begin{align*}
E &= E_{6061} \quad I = I_{AE} \\
\rho &= \rho_{6061} \quad \delta_e = c_{AE} \\
\sigma_y &= \sigma_y_{6061} \quad t = C_{AE}
\end{align*}
\]

static deflection
\[
\delta_{st} = \frac{P_{\text{driver}}b_{\text{driver}}}{9\sqrt{3}L_1E}\left(\frac{L_1}{L_1 - b_{\text{driver}}}\right)^2 = 0.071\text{ in}
\]

\[
F_{\text{driver}} = \left(1 + \sqrt{1 + \frac{2n_{\text{def}}h}{\delta_{st}}}\right)P_{\text{driver}} = 1152.1\text{ N}
\]

impact factor
\[
1 + \frac{2n_{\text{def}}h}{\delta_{st}} = 3.837
\]

\[
P_{\text{vehicle.self.weight}} = W_{\text{vehicle.self.weight}}L_1 = 155.7\text{ N}
\]

\[
x_{\text{vehicle.self.weight}} = \frac{L_1}{2} = 32\text{ in}
\]

Reality Check: total vehicle load.
\[
F_{\text{total.vehicle}} = \left(F_{\text{engine}} + n_{\text{shell}}F_{\text{shell.c}} + W_{\text{vehicle.self.weight}}L_1\right) = 120\text{ lbf}
\]

\[
F_{\text{total}} = F_{\text{total.vehicle}} + 2F_{\text{driver}} = 638\text{ lbf}
\]

Reactions to Wheel Connections

Sum of Moments about point A (distributed loads treated as point loads)
\[
R_B = \frac{1}{L_1}\left(1 + \frac{2n_{\text{def}}h}{\delta_{st}}\right)P_{\text{driver}} = 860.7\text{ N}
\]

Sum of forces in the y direction
\[
R_A = -R_B + F_{\text{driver}} + F_{\text{roll}} + F_{\text{engine}} + n_{\text{shell}}F_{\text{shell.c}} + P_{\text{vehicle.self.weight}} = 669.4\text{ N}
\]

Failure Due to Beam Bending (Norton 156)

Moment calculated at center of driver load to tack end of vehicle
\[
M_{\text{max}} = (P_B - F_{\text{shell.c}})(L_1 - x_{\text{engine}}) + (R_B - F_{\text{shell.c}} - F_{\text{engine}})(x_{\text{engine}} - x_{\text{roll}}) = 505.5\text{ N-m}
\]

\[
\sigma_{\text{bending}} = \frac{M_{\text{max}}c}{I} = 9.035 \times 10^7\text{ Pa}
\]

\[
N_{\text{bending}} = \frac{\sigma_y}{\sigma_{\text{bending}}} = 3
\]
Deflection

Assumptions:
Vehicle weight treated as uniform load
Driver treated as point load
Superposition used to find deflection at center of beam

\[
\delta_{\text{allowed}} := \frac{F_{\text{total, vehicle}}}{L_1} \cdot \frac{N}{m} = 328.363 \cdot \frac{N}{m}
\]

\[
\delta_{\text{uniform, dist}} := \frac{5w_{\text{uniform, dist}} \cdot L_1^4}{384 \cdot E \cdot I} = 0.08 \text{ in}
\]

\[
\delta_{\text{driver, center}} := \frac{F_{\text{driver}} \cdot b_{\text{driver}}}{48 \cdot E \cdot I} \left( \frac{3 \cdot L_1^2}{2} - 4 \cdot b_{\text{driver}}^2 \right) = 0.272 \text{ in}
\]

\[
\delta_{\text{max}} := \delta_{\text{driver, center}} + \delta_{\text{uniform, dist}} = 0.352 \text{ in}
\]

\[
N_{\text{deflection, 6061}} := \frac{\delta_{\text{allowed}}}{\delta_{\text{max}}} = 2.84
\]

Failure Due to Holes in Bearing (Tear out) (Norton 154)

\[
d_{\text{bolt}} := 10 \text{ mm} \quad \text{bolt diameter} \quad n_{\text{bolts}} := 2 \quad \text{number of bolts}
\]

\[
A_{\text{hole, bearing}} := d_{\text{bolt}} \cdot t = 0.026 \text{ in}^2
\]

\[
\sigma_{\text{bearing}} := \frac{R_B}{n_{\text{bolts}} \cdot A_{\text{hole, bearing}}} = 2.529 \times 10^7 \text{ Pa}
\]

\[
N_{\text{bearing, 6061}} := \frac{\sigma_y}{\sigma_{\text{bearing}}} = 10.7
\]

Failure Due to Bolts in Shear (Norton 162)

\[
A_{\text{shear}} := \pi \left( \frac{d_{\text{bolt}}}{2} \right)^2
\]

\[
\sigma_y, \text{bolt} := \sigma_y, 4340
\]

\[
\tau_{\text{bolts}} := \frac{4}{3} \frac{R_B}{n_{\text{bolts}} \cdot A_{\text{shear}}} = 7.306 \times 10^6 \text{ Pa}
\]

\[
N_{\text{shear, bolts}} := \frac{0.577 \cdot \sigma_y, \text{bolt}}{\tau_{\text{bolts}}} = 79
\]

Weight Calculations

\[
A_{\text{AE}, r} := \left( B_{\text{AE}, r} \cdot A_{\text{AE}, r} \right) - \left[ \left( B_{\text{AE}, r} - 2C_{\text{AE}, r} \right) \left( A_{\text{AE}, r} - 2 \cdot D_{\text{AE}, r} \right) \right] = 0.501 \text{ in}^2
\]

\[
\text{Weight per foot} := A_{\text{AE}, r} \cdot \rho \cdot g + 0.013 \frac{\text{lb ft}}{\text{ft}} = 0.602 \frac{\text{lb ft}}{\text{ft}}
\]

\[
\text{Weight one beam \ AE} := \text{Weight per foot} \cdot L_2 = 3.66 \text{ lb ft}
\]
2 Selected Material: 7075
Selected Shape: Tube

\[ E = E_{7075} \quad \rho = \rho_{7075} \quad \sigma_y = \sigma_y_{7075} \]
\[ L = L_{AE.7} \quad \delta = \delta_{AE.7} \]

Static deflection
\[ \delta = P_{\text{driver}} \frac{b_{\text{driver}}}{9 \sqrt{3} L_1 E_1} \left( \frac{L_1^2 - b_{\text{driver}}^2}{2} \right)^{\frac{3}{2}} = 0.27 \text{ in} \]

\[ F_{\text{driver}} = \left( 1 + \frac{2 \cdot \delta_{\text{def.}}}{\delta_{\text{st}}} \right) P_{\text{driver}} = 807.2 \text{ N} \]

\[ P_{\text{vehicle.self.weight}} = W_{\text{vehicle.self.weight}} L_1 = 155.7 \text{ N} \]
\[ x_{\text{vehicle.self.weight}} = \frac{L_1}{2} = 32 \text{ in} \]

Reality Check: total vehicle load.

\[ F_{\text{total.vehicle}} = (F_{\text{engine}} + n_{\text{shell}} F_{\text{shell.c}} + W_{\text{vehicle.self.weight}} L_1 - 2) = 120 \text{ lbf} \]
\[ F_{\text{total}} = F_{\text{total.vehicle}} + 2F_{\text{driver}} = 482.9 \text{ lbf} \]

Reactions to Wheel Connections
Sum of Moments about point A (distributed loads treated as point loads)

\[ R_B = \frac{\left( F_{\text{driver}} x_{\text{driver}} + F_{\text{roll}} x_{\text{roll}} + F_{\text{engine}} x_{\text{engine}} + P_{\text{vehicle.self.weight}} x_{\text{vehicle.self.weight}} \right)}{L_1} = 666.7 \text{ N} \]

Sum of forces in the y direction.

\[ R_B = -R_B + F_{\text{driver}} + F_{\text{roll}} + F_{\text{engine}} + n_{\text{shell}} F_{\text{shell.c}} + P_{\text{vehicle.self.weight}} = 518.6 \text{ N} \]

Failure Due to Beam Bending
Moment calculated at center of driver load to back end of vehicle

\[ M_{\text{max}} = (R_B - F_{\text{shell.c}})(L_1 - x_{\text{engine}}) + (R_B - F_{\text{shell.c}} - F_{\text{engine}})(x_{\text{engine}} - x_{\text{roll}}) + W_{\text{vehicle.self.weight}} (L_1 - x_{\text{driver}}) \frac{L_1 - x_{\text{driver}}}{2} = 395.2 \text{ N.m} \]

\[ \sigma_{\text{bending}} = \frac{M_{\text{max}} c}{I} = 1.642 \times 10^8 \text{ Pa} \]

\[ N_{\text{bending}} = \frac{\sigma_y}{\sigma_{\text{bending}}} = 3.1 \]
Deflection

Assumptions:
Vehicle weight treated as uniform load
Driver treated as point load
Superposition used to find deflection at center of beam

\[ \delta_{\text{unif.dist}} = \frac{F_{\text{total.vehicle}}}{L_1} = \frac{328.363 \text{ N}}{m} \]

\[ \delta_{\text{unif.dist}} = \frac{5w_{\text{unif.dist}}}{384 \cdot E \cdot I} = 0.306 \text{ in} \]

\[ F_{\text{driver}} = 807.204 \text{ N} \]

\[ \delta_{\text{driver.center}} = \frac{F_{\text{driver}} \cdot b_{\text{driver}}}{48 \cdot E \cdot I} \left(3 \cdot L_1^2 - 4 \cdot b_{\text{driver}}^2\right) = 0.725 \text{ in} \]

\[ \delta_{\text{max}} = \delta_{\text{driver.center}} + \delta_{\text{unif.dist}} = 1.031 \text{ in} \]

\[ \frac{\delta_{\text{allowed}}}{\delta_{\text{max}}} = 0.97 \]

Failure Due to Holes in Bearing

\[ d_{\text{bolt}} = 10 \text{ mm} \quad \text{bolt diameter} \quad n_{\text{bolts}} = 2 \quad \text{number of bolts} \]

\[ A_{\text{hole.bearing}} = d_{\text{bolt}} t = 0.026 \text{ in}^2 \]

\[ \gamma_{\text{bearing}} = \frac{R_B}{n_{\text{bolts}} A_{\text{hole.bearing}}} = 1.989 \times 10^7 \text{ Pa} \]

\[ \frac{\sigma_y}{\gamma_{\text{bearing}}} = 25.3 \]

Failure Due to Bolts in Shear

\[ A_{\text{shear}} = \frac{\pi \left(\frac{d_{\text{bolt}}}{2}\right)^2}{3} \]

\[ \tau_{\text{bolts}} = \frac{4}{3} \frac{R_B}{n_{\text{bolts}} A_{\text{shear}}} = 5.659 \times 10^6 \text{ Pa} \]

\[ \frac{0.577 \cdot \sigma_y \cdot \text{bolt}}{\tau_{\text{bolts}}} = 102 \]

Weight Calculations

\[ \text{Area}_{AE} = (B_{AE} \cdot A_{AE}) - \left[\left(B_{AE} - 2C_{AE}\right) \cdot (A_{AE} - 2 \cdot D_{AE})\right] = 0.329 \text{ in}^2 \]

\[ \text{Weight per foot} = \text{Area}_{AE} \cdot \rho \cdot g + 0.013 \text{ lbf/ft} = 0.414 \text{ lbf/ft} \]

\[ \text{Weight per beam}_{AE} = \text{Weight per foot} \cdot L_2 = 2.52 \text{ lbf} \]
3 Selected Material: Wood Fir
Selected Shape: Rectangle

\( E = E_{\text{fir}} \)
\( L = L_{\text{w}} \)
\( \rho = \rho_{\text{fir}} \)
\( c = c_{\text{w}} \)
\( \sigma_{\text{y,\ fir}} \)
\( t = b_{\text{w}} \)

Static deflection
\[ \delta_{\text{st}} = \frac{P_{\text{driver}} b_{\text{driver}}}{9 \sqrt{3} L_{1} E_{1} I_{1}} \left( \frac{L_{1} - b_{\text{driver}}}{2} \right)^{2} \approx 0.042 \text{ in} \]

\[ F_{\text{driver}} = \left( 1 + \frac{2 \cdot n_{\text{def}} - 1}{\delta_{\text{st}}} \right) \cdot P_{\text{driver}} = 1380.3 \text{ N} \]

\[ F_{\text{vehicle, self weight}} = W_{\text{vehicle, self weight}} \cdot L_{1} = 155.7 \text{ N} \]
\[ L_{1} = \frac{1}{2} = 32 \text{ in} \]

Reality Check: total vehicle load.
\[ F_{\text{total, vehicle}} = \left( F_{\text{engine}} + n_{\text{shell}} F_{\text{shell, c}} + W_{\text{vehicle, self weight}} \cdot L_{1} \right) \cdot 2 = 120 \cdot \text{lbf} \]
\[ F_{\text{total}} = F_{\text{total, vehicle}} + 2F_{\text{driver}} = 740.6 \text{ lbf} \]

Reactions to Wheel Connections
Sum of Moments about point A (distributed loads treated as point loads)
\[ R_{x} = \frac{F_{\text{driver}} x_{\text{driver}} + F_{\text{roll}} x_{\text{roll}} + F_{\text{engine}} x_{\text{engine}} + P_{\text{vehicle, self weight}} x_{\text{vehicle, self weight}} \ldots + F_{\text{shell, c}} x_{\text{shell, c}} + F_{\text{shell, c}} x_{\text{shell, c}} + F_{\text{shell, c}} x_{\text{shell, c}} + F_{\text{shell, c}} x_{\text{shell, c}}}{L_{1}} = 989.1 \text{ N} \]

Sum of forces in the y direction.
\[ R_{y} = -R_{x} + F_{\text{driver}} + F_{\text{roll}} + F_{\text{engine}} + n_{\text{shell}} F_{\text{shell, c}} + P_{\text{vehicle, self weight}} = 769.3 \text{ N} \]

Failure Due to Beam Bending
Moment calculated at center of driver load to back end of vehicle
\[ M_{\text{max}} = (R_{\text{B}} - F_{\text{shell, c}})(L_{1} - x_{\text{engine}}) + (R_{\text{B}} - F_{\text{shell, c}} - F_{\text{engine}})(x_{\text{engine}} - x_{\text{roll}}) \ldots = 583.5 \text{ N-m} \]
\[ + W_{\text{vehicle, self weight}}(L_{1} - x_{\text{driver}}) \cdot \frac{(L_{1} - x_{\text{driver}})}{2} \]

\[ \sigma_{\text{bending}} = \frac{M_{\text{max}} c}{I} = 1.48 \times 10^{7} \text{ Pa} \]

\[ N_{\text{bending}} = \frac{\sigma_{\text{y}}}{\sigma_{\text{bending}}} = 3 \]
Deflection

Assumptions:
Vehicle weight treated as uniform load
Driver treated as point load
Superposition used to find deflection at center of beam

\[ \delta_{\text{unif.dist.}} = \frac{F_{\text{total.vehicle}}}{L_1} = \frac{328.363 \text{ N}}{m} \]

\[ \delta_{\text{unif.dist.}} = \frac{5w_{\text{unif.dist.}} L_1^4}{384EI} = 0.048 \text{ in} \]

\[ \delta_{\text{driver.center}} = \frac{F_{\text{driver}} b_{\text{driver}}}{48EI} \left( \frac{3L_1^2}{2} - 4b_{\text{driver}}^2 \right) = 0.192 \text{ in} \]

\[ \delta_{\max} = \delta_{\text{driver.center}} + \delta_{\text{unif.dist.}} = 0.24 \text{ in} \]

\[ \frac{N_{\text{deflection.6061}}}{\delta_{\max}} = \frac{\delta_{\text{allowed}}}{\delta_{\max}} = 4.173 \]

Failure Due to Holes in Bearing

\[ d_{\text{bolt}} = 10 \text{ mm} \quad \text{bolt diameter} \quad n_{\text{bolts}} = 2 \quad \text{number of bolts} \]

\[ A_{\text{hole.bearing}} = d_{\text{bolt}}^2 = 0.394 \text{ in}^2 \]

\[ R_B = \frac{n_{\text{bolts}} A_{\text{hole.bearing}}}{2} = 1.947 \times 10^6 \text{ Pa} \]

\[ N_{\text{bearing.6061}} = \frac{\sigma_y}{\sigma_{\text{bearing}}} = 22.6 \]

Failure Due to Bolts in Shear

\[ A_{\text{shear}} = \pi \left( \frac{d_{\text{bolt}}}{2} \right)^2 \]

\[ \tau_{\text{bolts}} = \frac{4}{3} \frac{R_B}{n_{\text{bolts}} A_{\text{shear}}} = 8.396 \times 10^6 \text{ Pa} \]

\[ N_{\text{shear.bolts}} = \frac{0.577 \sigma_y \tau_{\text{bolts}}}{\tau_{\text{bolts}}} = 68.7 \]

Weight Calculations

Area := b_w h_w

Weight := Area \cdot \rho \cdot g = 0.82 \text{ lbf/ft} \]

Weight-per-foot := Weight \cdot L_2 = 4.99 \text{ lbf}
Selected Material: 4340

Selected Shape: Tube

\[ E_{4340} = E_{SE} \]
\[ \rho_{4340} = \rho_{SE} \]
\[ \sigma_{y,4340} = \sigma_{y,SE} \]
\[ t = t_{SE} \]

static deflection

\[ \delta_{def} = F_{driver} \cdot \left( \frac{L_1^2 - b_{driver}^2}{9 \sqrt{3} \cdot L_1 \cdot E \cdot I} \right)^{\frac{3}{2}} = 0.243 \text{ in} \]

\[ F_{driver} = \left( 1 + \frac{2 \cdot \delta_{def} \cdot b_{driver}}{\delta_{st}} \right) \cdot P_{driver} = 825.1 \text{ N} \]

Vehicle self-weight

\[ W_{vehicle, self-weight} \cdot L_1 = 155.7 \text{ N} \]

\[ l_{vehicle, self-weight} = \frac{L_1}{2} = 32 \text{ in} \]

Reality Check: total vehicle load.

\[ F_{total, vehicle} = (F_{engine} + n_{shell} \cdot F_{shell, c} + W_{vehicle, self-weight} \cdot L_1) \cdot 2 = 120 \text{ lbf} \]

\[ F_{total} = F_{total, vehicle} + 2F_{driver} = 491 \text{ lbf} \]

Reactions to Wheel Connections

Sum of Moments about point A (distributed loads treated as point loads)

\[ R_A = \frac{F_{driver} \cdot x_{driver} + F_{roll} \cdot x_{roll} + F_{engine} \cdot x_{engine} + P_{vehicle, self-weight} \cdot x_{vehicle, self-weight} \ldots}{x_{vehicle, self-weight} + F_{shell, c} \cdot x_{shell, 2} + F_{shell, c} \cdot x_{shell, 3} + F_{shell, c} \cdot x_{shell, 4}} = 676.8 \text{ N} \]

Sum of forces in the y direction.

\[ R_A = -R_B + F_{driver} + F_{roll} + F_{engine} + n_{shell} \cdot F_{shell, c} + P_{vehicle, self-weight} = 526.4 \text{ N} \]

Failure Due to Beam Bending

Moment calculated at center of driver load to back end of vehicle

\[ M_{max} = (R_B - F_{shell, c}) \cdot (L_1 - x_{engine}) + (R_B - F_{shell, c} - F_{engine}) \cdot (x_{engine} - x_{roll}) \ldots = 401.1 \text{ N \cdot m} \]

\[ + W_{vehicle, self-weight} \cdot (L_1 - x_{driver}) \cdot \frac{(L_1 - x_{driver})}{2} \]

\[ N_{bending} = \frac{M_{max} \cdot c}{I} = 3.092 \times 10^8 \text{ Pa} \]

\[ \sigma_{y, bending} = \frac{\sigma_y}{\sigma_{bending}} = 3.2 \]
Deflection

Assumptions:
Vehicle weight treated as uniform load
Driver treated as point load
Superposition used to find deflection at center of beam

\[ \delta_{\text{unif dist}} = \frac{F_{\text{total vehicle}}}{L_1} = 328.363 \, \text{in} \]

\[ \delta_{\text{unif dist}} = \frac{5w_{\text{unif dist}} L_1^4}{384 E I} = 0.276 \, \text{in} \]

\[ F_{\text{driver}} = 825.068 \, \text{N} \]

\[ \delta_{\text{driver}} = \frac{F_{\text{driver}} b_{\text{driver}}}{48 E I} \left( 3L_1^2 - 4b_{\text{driver}}^2 \right) = 0.667 \, \text{in} \]

\[ \delta_{\max} = \delta_{\text{driver center}} + \delta_{\text{unif dist}} = 0.943 \, \text{in} \]

\[ N_{\text{deflection.6061}} = \frac{\delta_{\text{allowed}}}{\delta_{\max}} = 1.06 \]

Failure Due to Holes in Bearing

\[ d_{\text{bolt}} = 10 \text{mm} \quad \text{bolt diameter} \]

\[ n_{\text{bolts}} = 2 \quad \text{number of bolts} \]

\[ A_{\text{hole bearing}} = d_{\text{bolt}}^4 = 0.026 \, \text{in}^2 \]

\[ \sigma_{\text{bearing}} = \frac{R_B}{n_{\text{bolts}} A_{\text{hole bearing}}} = 2.05 \times 10^7 \, \text{Pa} \]

\[ N_{\text{bearing.6061}} = \frac{\sigma_Y}{\sigma_{\text{bearing}}} = 48.8 \]

Failure Due to Bolts in Shear

\[ A_{\text{shear}} = \pi \left( \frac{d_{\text{bolt}}}{2} \right)^2 \]

\[ \tau_{\text{bolts}} = \frac{4}{3} \frac{R_B}{n_{\text{bolts}} A_{\text{shear}}} = 5.745 \times 10^6 \, \text{Pa} \]

\[ N_{\text{shear.bolts}} = \frac{0.577 \cdot \sigma_{Y \cdot \text{bolt}}}{\tau_{\text{bolts}}} = 100.4 \]

Weight Calculations

\[ A_{\text{SE}} = (B_{SE} \cdot A_{SE}) - \left[ (B_{SE} - 2C_{SE}) (A_{SE} - 2D_{SE}) \right] = 0.243 \, \text{in}^2 \]

\[ \text{Weight per foot} = \frac{\text{Area}_{\text{SE}} \cdot \rho \cdot g + 0.001 \, \text{lbf}}{\text{ft}} = 0.827 \, \text{lbf} \]

\[ \text{Weight per beam} \cdot L_2 = 5.03 \, \text{lbf} \]