Abstracts of Talks

30th Annual Workshop in Geometric Topology
Calvin College
June 13–15, 2013

Three Lectures on
"Riemannian Hyperbolization"

Pedro Ontaneda
Binghamton University

Abstract. Negatively curved Riemannian manifolds are fundamental objects in many areas of mathematics. But there were very few examples: apart from dimensions 4 and 6 every known negatively curved closed Riemannian manifold was homeomorphic to a hyperbolic one or homeomorphic to a branched cover of a hyperbolic one. If we allow singularities this situation changes because there is a rich and large class of negatively curved spaces that can be constructed using the Charney-Davis strict Hyperbolization process. (This process builds on the Hyperbolization process introduced by Gromov in 1987, which was later studied by Davis and Januszkiewicz). But the negatively curved manifolds contructed using the Charney-Davis strict hyperbolization process are far from being Riemannian because the metrics have large and highly complicated set of singularities. We will show how to remove these singularities, obtaining in this way a Riemannian strict hyperbolization process. Moreover we can do this Riemannian hyperbolization in a pinched way, that is, with curvature as close to -1 as desired.

In the first half of lecture I we will state the Main result and its corollaries. In the second half of lecture I and part of lecture II we will introduce three geometric processes: the two-variable warping trick (based on the Farrell-Jones warping trick), warp forcing and hyperbolic extensions. Also in lecture II we will discuss the construction of extremely useful differentiable structures: normal differentiable structures on cubical manifolds and on Charney-Davis hyperbolizations. Finally in lecture III we will sketch how to smooth metrics on hyperbolic cones.

Andrews-Curtis Moves Viewed as Biomorphic Events

Tony Bedenikovic
Bradley University

Abstract. The Andrews-Curtis conjecture is a long-standing conjecture regarding normal generating sets in free groups. Widely believed to be false, the conjecture suggests that any minimal normal generating set in a free group is equivalent to the free group's standard basis via elementary moves (inversion, multiplication, conjugation). I will review several positive results in the literature and mention an interesting generalization to arbitrary groups. The main purpose of the talk, however, is to describe work in progress to view Andrews-Curtis moves as organic, nature-mimicking events. Approaches to the conjecture inspired by this view will be discussed.

The Euler characteristic of a Haken 4-manifold

Allan Edmonds
Indiana University

Abstract. I will briefly introduce the class of aspherical manifolds known as Haken n-manifolds, developed by B. Foozwell and H. Rubinstein, and then outline a proof that the Euler characteristic of a closed Haken 4-manifold is non-negative.

The Lower Algebraic K-theory of Split Three-Dimensional Crystallographic Groups

Daniel Farley
Miami University

Abstract. We compute the lower algebraic K-theory of splt three-dimensional crystallographic groups. There are 73 such crystallographic groups in all, up to isomorphism, representing a third of the total number of three-dimensional crystallographic groups. I will attempt to sketch part of the classification, describe the classifying spaces that are used in the computation, and indicate how it is carried out in a representative example. (Joint work with Ivonne Ortiz.)

Witt space bordism

Greg Friedman
Texas Christian University

Abstract. We'll provide historical background on the computation of Witt bordism groups over $\mathbb{Q}$ (due to Siegel) and over other fields (due to the speaker) and conclude with an interesting open problem in characteristic 2.

Weak $\mathcal{Z}$-structures for some classes of groups

Craig Guilbault
University of Wisconsin–Milwaukee

Abstract. Motivated by the usefulness of boundaries in the study of δ-hyperbolic and CAT(0) groups, Bestvina introduced a general approach to group boundaries via the notion of a "$\mathcal{Z}$-structure" on a group G. Several variations on $\mathcal{Z}$-structures have been studied and existence results have been obtained for some very specific classes of groups. However, little is known about the general question of which groups admit any of the various $\mathcal{Z}$-structures, aside from the (easy) fact that any such G must have type F, i.e., G must admit a finite K$(G,1)$. In fact, Bestvina has asked whether *every* type F group admits a $\mathcal{Z}$-structure or at least a "weak" $\mathcal{Z}$-structure.

In this talk we will discuss some rather general existence theorems for weak $\mathcal{Z}$-structures. Among the main results are the following:

Theorem A. *If G is an extension of a nontrivial type F group by a nontrivial type F group, then G admits a weak $\mathcal{Z}$-structure.*

Theorem B. *If G admits a finite $K(G,1)$ complex K such that the G-action on $\widetilde{K}$ contains $1 \neq j \in G$ properly homotopic to $\mathrm{id}_{\widetilde{K}}$, then G admits a weak $\mathcal{Z}$-structure.*

Theorem C. *If G has type F and is simply connected at infinity, then G admits a weak $\mathcal{Z}$-structure.*

Busemann G-Spaces and Infinite Dimensions

Denise Halverson
Brigham Young University

Abstract. It is an unsolved problem whether or not all Busemann G-spaces are finite dimensional. Results towards this problem will be addressed.

Invariant contact structures on 7-dimensional nilmanifolds

Sergii Kutsak
University of Florida

Abstract. I will show how to find all 7-dimensional nilmanifolds that admit an invariant contact structure

Isometric Embeddings of Polyhedra

Barry Minemyer
Binghamton University

Abstract. In 1954–1956 John Nash solved the isometric embedding problem for Riemannian Manifolds. Some of his results were extended to manifolds endowed with an indefinite metric tensor by Greene and Gromov/Rokhlin independently in 1970. In this talk we will discuss combinatorial analogues of these results to polyhedra.

The fattened Davis complex and weighted L^2-(co)homology of non-right angled Coxeter groups

Wiktor Mogilski
University of Wisconsin–Milwaukee

Abstract. Associated to a Coxeter system (W, S) there is a contractible simplicial complex Σ called the Davis complex. Given a positive real multiparameter q, one can define the weighted L^2-(co)homology groups of Σ. We propose a program to compute the weighted L^2-(co)homology of by considering a fattened version of this complex. The program proves successful provided that one can understand the weighted L^2-(co)homology of nonspherical special subgroups of W. We then restrict our attention to Coxeter groups with $m_{st} = 3; \infty$ and apply this method to perform some computations.

Affine maps between CAT(0) Spaces

Chris Mooney
Bradley University

Abstract. To what extent do the geodesics in a space determine the geometry of the space? To study this question, we investigate "affine maps" – continuous functions between metric spaces which preserve geodesics and rescale linearly along each. Classical work, culminating with a recent Theorem of Lytchak completely classifies all possible affine functions from Riemannian manifolds into metric spaces.

In the CAT(0) setting, an affine map between CAT(0) spaces determines a real-valued function of the boundary, which behaves very nicely with respect to both the visual topology and Tits metric. We use this to classify affine maps of CAT(0) spaces admitting a geometric group action.

(Joint work with Hanna Bennett and Ralf Spatzier.)

Finite dimensionality of $\mathcal{Z}$-boundaries and its consequences

Molly Moran
University of Wisconsin–Milwaukee

Abstract. The rich study of boundaries of CAT(0) and hyperbolic groups led M. Bestvina to formalize the concept of a group boundary by defining a $\mathcal{Z}$-structure on a group. In his original definition, a $\mathcal{Z}$-structure on a group G is a pair of spaces $(\hat{X}, Z)$ where $\hat{X}$ is a compact ER, Z is a $\mathcal{Z}$-set in $\hat{X}$, G acts freely, cocompactly, and properly on $X = \hat{X} - Z$ and the collection of G-translates of a compact set in X forms a null sequence in $\hat{X}$. In this setting, Z is finite dimensional. We show that this result can be extended to the case that $\hat{X}$ is an AR, that is when $\hat{X}$ need not be finite dimensional. We also explore results that can be obtained by knowing the $\mathcal{Z}$-boundary is finite dimensional.

Patterson-Sullivan Theory in CAT(0) Spaces

Russell Ricks
University of Michigan

Abstract. Patterson's construction of a certain probability measure, called the Patterson-Sullivan measure, on the limit set of a Fuchsian group has been generalized and used in a number of geometric settings. One application is to construct an invariant probability measure, called the Bowen-Margulis measure, on the geodesic flow. We will discuss the setting of CAT(0) spaces, and recover some information about the space itself.

Extensions for G-acyclic resolutions

Leonard R. Rubin
University of Oklahoma

Abstract. Given an abelian group $G, n \in \mathbb{N}$, and a compact metrizable space X with $\dim_G(X) \leq n$, a G-acyclic resolution is a surjective map $\pi : Z \to X$ having fibers which are of trivial Čech cohomology with respect to G. We always want Z to be compact and metrizable; in some cases we may require that $\dim(Z) \leq n$. In case $G = Z/p$, proofs of the existence of such data have required complicated extensions over triangulated polyhedra. We seek proofs in this case and perhaps others in which one may trade such difficult extensions with some facts in general topology, thereby making the proofs accessible to a wider class of readers.

This is work in progress jointly with Vera Tonić.

The Atiyah conjecture for virtually compact special groups

Kevin Schreve
University of Wisconsin–Milwaukee

Abstract. If Y is a manifold with a cocompact proper G-action, Atiyah asked about possible values for the L^2-Betti numbers of Y. In particular, he asked whether they were always rational, or even integers if G is torsion free. If H satisfies the Atiyah conjecture, and G is a finite group extension of H, we will give conditions on H for the conjecture to lift to G. We will also show that compact special groups, as introduced by Haglund and Wise, satisfy these conditions.

Hat spines and splittings

Pete Sparks
University of Wisconsin-Milwaukee

Abstract. We are interested in contractible manifolds M^n which decompose or **split** as $M^n \approx A \cup_C B$ where $A, B, C \approx \mathbb{R}^n$ or $A, B, C \approx \mathbb{B}^n$. We introduce a countable collection of 4-manifolds, $\{M_i : i = 1, 2, ...\}$, each containing a spine which can be written as $A \cup_C B$ with A, B, and C all collapsible. This implies each M_i splits as $\mathbb{B}^4 \cup_{\mathbb{B}^4} \mathbb{B}^4$. Using sequences from this collection we form infinite boundary connect sums whose interiors each split as $\mathbb{R}^4 \cup_{\mathbb{R}^4} \mathbb{R}^4$. We thus obtain an uncountable collection of contractible open 4-manifolds which split in this way.

The Tits Alternative For Generalized Baumslag-Solitar Groups

Mathew Timm
Bradley University

Abstract. A generalized Baumslag-Solitar group is a group that has a graph of groups description in which all vertex and edge groups are copies of the integers. It is know that this class of groups satisfies the Tits Alternative. We give a topological proof of this fact.

Relations between various boundaries of relatively hyperbolic groups

Hung Tran
University of Wisconsin–Milwaukee

Abstract. Suppose a group G is relatively hyperbolic with respect to a collection $\mathbb{P}$ of its subgroups and also acts properly, cocompactly on a $CAT(0)$ (or δ–hyperbolic) space X. The relatively hyperbolic structure provides a relative boundary $\partial(G, \mathbb{P})$. The $CAT(0)$ structure provides a different boundary at infinity ∂X. In this article, we examine the connection between these two spaces at infinity. In particular, we show that $\partial(G, \mathbb{P})$ is G–equivariantly homeomorphic to the space obtained from ∂X by identifying the peripheral limit points of the same type.

Riemannian manifolds with nontrivial symmetry

Wouter Van Limbeek
University of Chicago

Abstract. In this talk I will discuss the problem of classifying all closed Riemannian manifolds whose universal cover has nondiscrete isometry group. Farb and Weinberger solved this under the additional assumption that M is aspherical: roughly, they proved M is a fiber bundle with locally homogeneous fibers. However, if M is not aspherical, many new examples and phenomena appear. I will exhibit some of these, and discuss progress towards a classification.