Three Lectures on
"Riemannian Hyperbolization"

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Abstract. Negatively curved Riemannian manifolds are fundamental objects in many areas of mathematics. But there were very few examples: apart from dimensions 4 and 6 every known negatively curved closed Riemannian manifold was homeomorphic to a hyperbolic one or homeomorphic to a branched cover of a hyperbolic one. If we allow singularities this situation changes because there is a rich and large class of negatively curved spaces that can be constructed using the Charney-Davis strict Hyperbolization process. (This process builds on the Hyperbolization process introduced by Gromov in 1987, which was later studied by Davis and Januszkiewicz). But the negatively curved manifolds contructed using the Charney-Davis strict hyperbolization process are far from being Riemannian because the metrics have large and highly complicated set of singularities. We will show how to remove these singularities, obtaining in this way a Riemannian strict hyperbolization process. Moreover we can do this Riemannian hyperbolization in a pinched way, that is, with curvature as close to -1 as desired.

In the first half of lecture I we will state the Main result and its corollaries. In the second half of lecture I and part of lecture II we will introduce three geometric processes: the two-variable warping trick (based on the Farrell-Jones warping trick), warp forcing and hyperbolic extensions. Also in lecture II we will discuss the construction of extremely useful differentiable structures: normal differentiable structures on cubical manifolds and on Charney-Davis hyperbolizations. Finally in lecture III we will sketch how to smooth metrics on hyperbolic cones.