

Weak \mathcal{Z} -structures for some classes of groups

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Abstract. Motivated by the usefulness of boundaries in the study of δ -hyperbolic and CAT(0) groups, Bestvina introduced a general approach to group boundaries via the notion of a “ \mathcal{Z} -structure” on a group G . Several variations on \mathcal{Z} -structures have been studied and existence results have been obtained for some very specific classes of groups. However, little is known about the general question of which groups admit any of the various \mathcal{Z} -structures, aside from the (easy) fact that any such G must have type F, i.e., G must admit a finite $K(G, 1)$. In fact, Bestvina has asked whether *every* type F group admits a \mathcal{Z} -structure or at least a “weak” \mathcal{Z} -structure.

In this talk we will discuss some rather general existence theorems for weak \mathcal{Z} -structures. Among the main results are the following:

Theorem A. *If G is an extension of a nontrivial type F group by a nontrivial type F group, then G admits a weak \mathcal{Z} -structure.*

Theorem B. *If G admits a finite $K(G, 1)$ complex K such that the G -action on \tilde{K} contains $1 \neq j \in G$ properly homotopic to $\text{id}_{\tilde{K}}$, then G admits a weak \mathcal{Z} -structure.*

Theorem C. *If G has type F and is simply connected at infinity, then G admits a weak \mathcal{Z} -structure.*