On the Singer conjecture for Coxeter groups

Tim Schroeder
University of Wisconsin-Milwaukee

Abstract. Given any Coxeter system (W, S), there is an associated CW-complex, $\Sigma(W, S)$ (or simply Σ), on which W acts properly and co-compactly. When the nerve L of (W, S) is a triangulation of an $(n-1)$-sphere, Σ is an n-manifold. There is a variation of the Singer conjecture for Coxeter groups, i.e. that when L is a sphere, the reduced ℓ^2-homology of Σ vanishes in all but the middle dimension. We prove this in the case L is a 2-sphere by applying a theorem of Andreev, which gives the conditions under which a Coxeter group acts as a classical reflection group on $\mathbb{H}^3$. We then prove that this result implies the case when (W, S) is an even Coxeter system and L is a flag triangulation of $\mathbb{S}^3$.