Conditions under which a sewing of crumpled n-cubes yields S^n

Robert Daverman
University of Tennessee

Abstract. A *crumpled n-cube* is the closure of the complement of an n-cell wildly embedded in S^n, and a *sewing* of crumpled n-cubes is a homeomorphism between their boundaries. Associated with any such sewing $h : \text{Bd } C_1 \to \text{Bd } C_2$ is the *sewing space* $C_1 \cup_h C_2$, namely the quotient obtained from the disjoint union of C_1, C_2 under identification of each $x \in \text{Bd } C_1$ with $h(x) \in \text{Bd } C_2$. It is known that $C_1 \cup_h C_2$ is an n-manifold (and, hence, S^n) if it satisfies the following Strong Mismatch Property: any two maps $f_i : I^2 \to C_i$ can be approximated, arbitrarily closely, by maps $F_i : I^2 \to C_i$ such that $F_2(I^2) \cap h(F_1(I^2) \cap \text{Bd } C_1) = \emptyset$. A related Weak Mismatch Property, which is known to be a necessary condition for $C_1 \cup_h C_2$ to be a manifold when $n \geq 5$, allows the approximations F_i to range into S^n instead of being restricted to C_i. Theorem: In case C_1, C_2 are crumpled n-cubes satisfying the Disjoint Disks Property, then the Weak Mismatch Property is a necessary and sufficient condition for $C_1 \cup C_2$ to be S^n. Also discussed will be an Intermediate Property and sharp conditions under which a sewing with this property yields S^n.