PROBLEM SESSION

1. **Lawrence Brenton**
   (a) Let X be the cone on a homology 3-sphere M. Does there exist a Lorentzian metric g on X that is homogeneous on cross sections such that (X, g) satisfies the dominant energy condition?
   (b) If "no," where does the obstruction lie?
   (c) Will the spacetimes of part (a) always recollapse in a "big crunch," or does this depend on the choice of metric?

2. **Robert Daverman**
   (a) If X is a compact ANR homology 3-manifold, does there exist a real 3-manifold M such that M is homotopy equivalent to X?
   (b) If so, does X embed in $M \times \mathbb{R}$?
   (c) If so, is $X \times \mathbb{R} \cong M \times \mathbb{R}$?

3. **David Wright**
   Are there examples of compact 3-manifolds (or n-manifolds) in which every homeomorphism is isotopic to the identity?

4. **Tadek Dobrowolski**
   Let X be a contractible, locally contractible compact metric space. Does X have the fixed point property?
   The answer is known to be "yes" if there exists a function $\lambda : X \times X \times [0, 1] \to X$ such that

$$\lambda(x, y, 0) = x,$$
$$\lambda(x, y, 1) = y, \text{and}$$
$$\lambda(x, x, t) = x \text{ for } 0 \leq t \leq 1.$$

   Every AR has such a function.

5. **Steve Ferry**
   Is there a sequence of Riemannian manifolds, sharing a fixed contractibility function, that approach (in Gromov-Hausdorff space) an infinite dimensional space with a bound on volume?
   Definitions: A *contractibility function* on M is a function $\rho : (0, \infty) \to (0, \infty)$ such that for every $t > 0$ and for every $x \in M$

the ball of radius t in M centered at x is contractible in the ball of radius $\rho(t)$. If X and Y are compact metric spaces, the *Gromov-Hausdorff distance* $d_{\mathrm{GH}}(X, Y)$ is defined by

$$d_{\mathrm{GH}}(X,Y) = \inf\left\{ d^Z(X,Y) \;\middle|\; Z^{\text{metric space}} \supset X, Y \right\},$$

where d^Z is the usual Hausdorff distance between subcompacta of Z.

6. **Craig Guilbault**
Given a homomorphism $\mu : G \to \pi_1(M)$, with G a finitely generated group and M a closed manifold, such that $\ker(\mu)$ is perfect, does there exist a 1-sided h-cobordism that realizes μ? In other words, does there exist a triple (W, M, M^*) of manifolds such that $\partial W = M \sqcup M^*$, $M \hookrightarrow W$ is a homotopy equivalence, and

$$\begin{array}{ccc} \pi_1(M^*) & \longrightarrow & \pi_1(W) \\ \approx\uparrow & & \uparrow\approx \\ G & \xrightarrow{\ \mu\ } & \pi_1(M) \end{array}$$

commutes? [This is the reverse of Quillen's +-construction.]

7. **Sasha Dranishnikov**
   (a) Is $\operatorname{asdim}(X) = \dim(\nu X)$?
   (b) If Γ is a CAT(0) group, is $\operatorname{asdim}(\Gamma) < \infty$?
   (c) For $n \geq 2$, does there exist a Coxeter group Γ such that $\operatorname{vcd}_{\mathbb{Q}} \Gamma = 2$ and $\operatorname{vcd}_{\mathbb{Z}} \Gamma = n$?