Math 355 Homework Problems #3

MATRIX ANALYSIS AND APPLIED LINEAR ALGEBRA, by C. Meyer

1. For the matrix

$$A = \begin{pmatrix} 2 & 7 & 3 & 4 \\ 1 & 1 & -1 & -3 \\ 4 & 6 & -2 & -8 \end{pmatrix},$$

find a basis for $\mathrm{Col}(\boldsymbol{A})$, $\mathrm{Col}(\boldsymbol{A}^{\mathrm{H}})$, $\mathrm{Null}(\boldsymbol{A})$, $\mathrm{Null}(\boldsymbol{A}^{\mathrm{H}})$.

2. For the matrix and vector,

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & -1 & -1 \\ 3 & -5 & 4 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 5 \\ -7 \\ 3 \end{pmatrix},$$

write $\boldsymbol{b} = \boldsymbol{b}_{\mathrm{c}} + \boldsymbol{b}_{\mathrm{n}}$, where $\boldsymbol{b}_{\mathrm{c}} \in \mathrm{Col}(\boldsymbol{A})$ and $\boldsymbol{b}_{\mathrm{n}} \in \mathrm{Null}(\boldsymbol{A}^{\mathrm{H}})$.

3. Suppose,

$$X = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \right\}, \quad Y = \mathrm{Span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

We have $\mathbb{F}^3 = X \oplus Y$.

(a) Find the projection matrix, $\boldsymbol{P}_Y$, that corresponds to the projection onto Y.

(b) Find the projection matrix, $\boldsymbol{P}_X$, that corresponds to the projection onto X.

4. Fit the data, $(1,0), (2,1), (3,2), (4,4), (5,6)$, to the line, $y = a_0 + a_1 x$, using the normal equations. What are a_0, a_1?

5. Here you will *numerically* solve the normal equations to fit curves to data. The use of MATLAB will be necessary. There are tutorials in the shared Google Drive folder. The data is also located in that folder. Import the data from the file, "`LeastSquaresDataQuadratic.mat`". You will see two variables, x and y. Use least-squares to fit the data with the quadratic function $y = a_0 + a_1 x + a_2 x^2$. What are a_0, a_1, a_2? Plot the data, along with the least-squares curve, on one plot.

6. Consider the **consistent** linear system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$. Suppose that $\boldsymbol{y} \in \mathrm{Null}(\boldsymbol{A}^{\mathrm{H}})$. Show that $\boldsymbol{y}^{\mathrm{H}}\boldsymbol{b} = 0$. (this result is known as the Fredholm alternative)