# Math 355 Homework Problems #0

**Notation.** In all that follows, $\mathbb{F} = \mathbb{R}$, or $\mathbb{C}$. The notation is used when it makes no difference if the numbers are real-valued or complex-valued. The symbol, $\mathcal{M}_{m\times n}(\mathbb{F})$, represents the set of all matrices with $m$ rows and $n$ columns where the entries are in $\mathbb{F}$. The symbol, $\mathbb{F}^n$, represents the set of all vectors of length $n$ with entries in $\mathbb{F}$.

**1.** Consider the homogeneous system $\boldsymbol{Ax} = \boldsymbol{0}$, where $\boldsymbol{A} \in \mathcal{M}_{m\times n}(\mathbb{F})$ with $m < n$. Explain why this system must always have an infinite number of solutions.

**2.** Consider the nonhomogeneous system $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A} \in \mathcal{M}_{m\times n}(\mathbb{F})$ and $\boldsymbol{b} \in \mathbb{F}^m$ is nonzero.

(a) If $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are two solutions, must it be the case that $3\boldsymbol{x}_1 - 4\boldsymbol{x}_2$ is also a solution? Why, or why not?

(b) Suppose that $m \geq n$, and further suppose that the system is consistent. What must the row-reduced matrix look like if the solution is unique?

**3.** Find all of the solutions to the system

$$\begin{aligned} x - 3y - 4z &= -6 \\ 2x + 4z &= -6 \\ -6x + 4y + 4z &= 22. \end{aligned}$$

If the system is not consistent, state why.

**4.** Find eigenvalues and associated eigenvectors for the following matrices:

(a) $\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 0 \\ -6 & 5 & 0 \\ 3 & 4 & 7 \end{pmatrix}$