Find RREF of augmented matrix (A|b)

```
A = sym([1,2,3;4,5,6;7,8,9])
```

A =

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

```
b = sym([10;11;12])
```

b =

$$\begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$$

```
Ab = horzcat(A,b)
```

Ab =

$$\begin{pmatrix} 1 & 2 & 3 & 10 \\ 4 & 5 & 6 & 11 \\ 7 & 8 & 9 & 12 \end{pmatrix}$$

```
rref(Ab)
```

ans =

$$\begin{pmatrix} 1 & 0 & -1 & -\frac{28}{3} \\ 0 & 1 & 2 & \frac{29}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Find eigenvalues and eigenvectors symbolically

```
[vA,eAt] = eig(A)
```

vA =

$$\begin{pmatrix} 1 & -\frac{3\sqrt{33}}{22} - \frac{1}{2} & \frac{3\sqrt{33}}{22} - \frac{1}{2} \\ -2 & \frac{1}{4} - \frac{3\sqrt{33}}{44} & \frac{3\sqrt{33}}{44} + \frac{1}{4} \\ 1 & 1 & 1 \end{pmatrix}$$

eAt =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{15}{2} - \frac{3\sqrt{33}}{2} & 0 \\ 0 & 0 & \frac{3\sqrt{33}}{2} + \frac{15}{2} \end{pmatrix}$$

```
eA = diag(eAt)
```

eA =

$$\begin{pmatrix} 0 \\ \frac{15}{2} - \frac{3\sqrt{33}}{2} \\ \frac{3\sqrt{33}}{2} + \frac{15}{2} \end{pmatrix}$$

```
l2A = eA(2)
```

l2A =

$$\frac{15}{2} - \frac{3\sqrt{33}}{2}$$

```
v2A = vA(:,2)
```

v2A =

$$\begin{pmatrix} -\frac{3\sqrt{33}}{22} - \frac{1}{2} \\ \frac{1}{4} - \frac{3\sqrt{33}}{44} \\ 1 \end{pmatrix}$$

Find eigenvalues and eigenvectors numerically

```
[vAn,eAnt] = eig(vpa(A))
```

vAn =

$$\begin{pmatrix} 0.23197068724628586166084715996366 & -0.40824829046386301636621401245098 & 0.785 \\ 0.52532209330123369343198799512899 & 0.81649658092772603273242802490196 & 0.086 \\ 0.81867349935618152520312883029432 & -0.40824829046386301636621401245098 & -0.61 \end{pmatrix}$$

eAnt =

$$\begin{pmatrix} 16.116843969807042989775917202328 & 0 & \\ 0 & 5.6783944246442181604146204156927 \, 10^{-39} & \\ 0 & 0 & -1.1 \end{pmatrix}$$

```
eAn = diag(eAnt)
```

eAn =

$$\begin{pmatrix} 16.116843969807042989775917202328 \\ 5.6783944246442181604146204156927\ 10^{-39} \\ -1.1168439698070429897759172023284 \end{pmatrix}$$

```
l3An = eAn(1)
```

l3An = $16.116843969807042989775917202328$

```
v3An = vAn(:,1)
```

v3An =

$$\begin{pmatrix} 0.23197068724628586166084715996366 \\ 0.52532209330123369343198799512899 \\ 0.81867349935618152520312883029432 \end{pmatrix}$$

Find eigenvalues and eigenvectors, and scale eigenvector to have sum 1

```
B = sym([1/10,2/10,3/10;4/10,5/10,6/10;5/10,3/10,1/10])
```

B =

$$\begin{pmatrix} \frac{1}{10} & \frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{1}{2} & \frac{3}{5} \\ \frac{1}{2} & \frac{3}{10} & \frac{1}{10} \end{pmatrix}$$

```
[vB,eBt] = eig(B);
eB = diag(eBt)
```

eB =

$$\begin{pmatrix} 0 \\ -\frac{3}{10} \\ 1 \end{pmatrix}$$

```
l3B = eB(3);
v3B = vB(:,3);
v3Bs = v3B/sum(v3B)
```

v3Bs =

$$\begin{pmatrix} \frac{27}{130} \\ \frac{33}{65} \\ \frac{37}{130} \end{pmatrix}$$

```
vpa(v3Bs,12)
```

ans =

$$\begin{pmatrix} 0.207692307692 \\ 0.507692307692 \\ 0.284615384615 \end{pmatrix}$$